
THE GENTLEMAN'S
MATHEMATICAL COMPANION,
FOR THE
YEAR 1799.

ANSWERS TO THE LAST YEAR'S ENIGMAS.

- | | | | |
|------------|----------|-----------------|----------------------|
| 1. BELL | 4. DEATH | 7. PORTER | 10. WEAVER'S SHUTTLE |
| 2. LEATHER | 5. EARTH | 8. YESTERDAY | 11. TIME |
| 3. STONE | 6. DEW | 9. STAGE WAGGON | 12. Prize, WISDOM. |

Answers to the Prize Enigma.

1. *By the Rev. J. Shackleton, Thornton, near Bradford, Yorkshire.*

Nature, and nature's laws, lay hid in night,
Till Newton's *Wisdom* brought the same to light.

2. *By Mr. Benjamin Kemp, Farnsfield.*

Ingenious Bosworth writes in themes divine,
Whilst pious *Wisdom* flows thro' every line.

3. *By Mr. Thomas Nield, Master of Overton Boarding School.*

Hail, balmy *Wisdom*, guider of all right,
May thou be ne'er debarred from my fight;
Then what I do, if guided but by thee,
Will be what's right, and I from wrong am free.

4. *The Wish, by Mr. R. Humber, Brighton.*

Grant ine, ye hallow'd skies, that grace divine,
That thro' each line the prize may shine;
Wisdom attend my steps—adorn my ways,
And mercy beam to gild my fleeting days:
At last may I, cloth'd in a Saviour's robe,
Obtain admission in yon blest abode.

5. *Address to Mr. Bosworth, by J. Hindson, Lincoln.*

Hail, tuneful bard, thy verses charm the ear,
Thy worth and learning are well known to fame;
Long may thy lore, in *Wisdom's* page appear,
A grateful age will then esteem thy name.

6. *Joshua's Resolution*, by Mr. J. Savage, Norton, near Tow-
chester.

O, 'twas a noble resolution, made
By Joshua, Israel's captain, when he said,
Whatever others may do, I, said he,
Will serve the Lord, with all my family,
Or would the master of each house engage,
In Joshua's vow, in this enlightened age,
Thousands would then be led in *Wisdom's* way,
Who, uncontroul'd, in folly's path now stray.
Were this the case, then would our mourning Isle,
From trouble free, in peace and plenty smile;
No more domestic quarrels would appear,
Nor we no foreign enemy need fear;
For O, that gracious all-sufficient power!
Who was to Israel a safe refuge tower,
Would surely guard us with peculiar care,
Did we but trust in him with heart sincere.

7. *By Mr. William Robinson, London.*

Wisdom! thou chiefest good, may I ever share
Thy influence, to lessen corroding care;
And may my steps to thee for e'er incline,
That other virtues may in me combine.

8. *Address to Youth*, by Mr. T. Woolston, Adderbury.

Hearken, O youth, my counsels now receive,
And taste the numerous blessings I can give;
Not the choice silver, nor the purer gold,
Can equal those delights which I unfold:
I give man prudence, and superior skill,
To search out truth, and mould the erring will.
By me establish'd, mighty kings shall reign,
While pride and arrogance shall rage in vain;
Those who shall seek my gifts with ardent love,
Shall surely find, and all my riches prove.
Would you great nature's choicest secrets know,
And trace them in the skies, or depths below;
I, *Wisdom*, teach the soaring mind to climb,
And gain the height of knowledge most sublime;
Unlock the air, and of the depths beneath,
And shew the almighty power in all that breathe;
Whether rich providence around thee pours
Abundant wealth, or poverty still lours;
Guided by me, thy steps shall never swerve,
But providence shall in thy ways preserve;
True pleasures there, on every side shall rise,
And raise thy mind to taste superior joys;
Thy life shall flourish, all thy stores increase,
For all my paths are pleasantness and peace.

II. No. II. THE MATHEMATICAL COMPANION.

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Other answers were also given by Messrs. Aristarchus, Adams, Anderson, Allison, Airs, Brown, Burton, Bostoniensis, Bainbridge, Cunliff, Carpenter, Cicero, Davinso, Dawson, Denby, Ely, Evans, Everett, Farmer, Forster, Griffith, Gruchy, Guy, Guernsey, Gilbert, Hill, Howard, Hartley, Knowles, Lambert, Lisle, M'Arthur, Norton, Norman, Orpheus, Penry, Robinson, Sarrat, Surlees, Thompson, Thoubren, Tompkins, Turner, Watson, Wilson, and Young.

GENERAL ANSWERS TO THE ENIGMAS

1. Overton Club, by Mr. Thomas Nield, Master of a Boarding School, Overton, near Wrexham.

Had I but time, and talents to indite, 11
 I often would unto friend Davis write;
 But having little, now that mite embrace,
 So print my story, if it's worth a place.
 Each Friday night, a set of jovial blades,
 Of different callings, and of different trades,
 Do meet, like friends, to pass the time away,
 And drink good porter, till they all get gay. 7
 Here's Doctor Shuttle, with his box in hand, 10
 The club alone his wisdom will withstand, Pr.
 Points out the lines, how battles might be won,
 How this thing so, and that should have been done.
 Each then attack him, and raise such a jargon,
 Just like the rattling of some noisy waggon: 9
 Another friend, a dealer much in leather, 2
 Sometimes breaks loose and over-runs his tether;
 Pray, give me leave, he cries, my learned friend,
 And my few words will soon the contest end.
 The Grocer, then, he cries, the fact I'll tell,
 And from his pocket draws, and rings a bell, 1
 Till drops of sweat, like dew, run down his face; 6
 He states, and talks, and tries to clear the case.
 When all is o'er, we sit, drink, chat again;
 Say, yesterday, how many souls were slain; 8
 The earth and stone, how dy'd with human blood, 5 3
 How Duncan death in all its forms withstood; 4
 Drink to each other some good, loyal toast,
 And of the valor of our country boast:
 Then pay our score, shake hands, and part good friends;
 So now I've done, and thus my story ends.

2. On Winter. By Mr. J. Hindson, Lincoln.

Swift as a shuttle Phæbus turns his eyes, 10
 His glimm'ring rays dart feebly from the west;
 The earth around in rude confusion lies, 5
 The awful change by every one's confest. 4 Death

Nature of late was pleasing to our view,
 And fanning zephyrs prest thro' the air;
 Each *day* the fields were tipt with gentle *dew*,
 And *cloath'd* in richest green they did appear.
 The grateful lark then hail'd the rosy morn,
 The chirping songsters *tun'd* their various lays;
 The hind secured his store of hay and corn,
 For which, eternal *wisdom* he does praise;
 Past are those *times* and different scenes succeed,
 Keen and severe the North wind fiercely blows;
 The stream *forgets* its wonted course to keep,
 Fix'd like a mass of *stone*, no longer flows.
 Cold drizzling snow, and storms of hail and rain,
 United roar, and rage in the air;
 Winter extends her desolating reign,
 And fades the blighted beauty of the year.
 The same the case when health and vigour flows,
 And blooming youth beats high in ev'ry vein;
 Age and his wrinkles lay our glory low,
 Swift as a *waggon* gets by us unseen.

8, 6
2, Leather

1, Bell

Pr.

11

7, Porter

3

9

3. By Master Thomas Thoubren, Student, at Mr. Rutherford's
 Academy, Lanchester, Durham.

Ah! *death*, with all thy darts of steel,
 Who can withstand thy pow'r?
 As quick as *lightning* thou dost run,
 To cut the tender flow'r.
 When a commission thou'lt receiv'd,
 To any of our race;
 No *waggon*, *shuttle*, nor no *stone*,
 Can fly with equal pace.
 Our *time* on *earth* is like the *dew*
 That fadeth with the sun;
 A few more fleeting breaths we draw,
 Then all our work is done.
 Oh! then let me true *wisdom* have,
 While Christ with mankind strives;
 That when the final summons comes,
 I may ascend the skies,
 Then let my body go to dust;
 My shroud of *leather* be;
 While the 'larm *bell* begins to knell,
 The *Porters* mourn for me.
 Dry up your tears, mourn for yourselves,
 E're Satan you deceive;
 That you may make your peace with God,
 And ever with him live.

4

8. Yesterday.

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11 5 6

Pr.

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4. *A Winter Scene. A general Answer to the Enigmas, Charades, and Rebusses. By Mr. B. Kemp, Farnsfield, Nottinghamshire.*

Stern winter now with hoary triumph reigns;
 No more the *Lambkins* skip on dewy plains;
Hark, through the welkin, how the frigid breeze,
 Impetuous whistles thro' the leafless trees.
 While Sol's resplendent beams scarce deign to glow,
 To glad the *Earth*, or melt the drifted snow;
 The *woodlark*, *linnet*, and *canary's* notes,
 Are silent all—no *musick* swells their throats.
 Slowly the *waggon* does from *Brighthelmstone* steer,
 The rugged roads impede their swift career;
 The busy seamen now the *cockboat* hies,
 To gain from off the shores their merchandize.
 Whilst *Humber*, *Jervis*, *Campbell*, may retire,
 And pass the *day* around the social fire.
 Prompted by nature, each a covert seeks,
 This *wisdom* teaches, and thus reason speaks.
 But oh, when thus the chilling tempests roar,
 And rush with fury thro' the cottage door;
 Where famished babes cling round a weeping fire,
 And lisp in vain for raiment, food, and fire;
 Cold and benumb'd, behold their wretched plight,
 No friendly *bedstead* cheers their limbs at night;
 Hear this, ye proud and insolently great,
 Who loll supinely on a bed of state;
 Or, at some *wine vaults* spend your *time* and wealth,
 Corrupting morals, and impairing health.
 Think, while you feast on each luxuriant meal,
 What hungry, uncloth'd wretchedness must feel.
 Perhaps your gates are barred against the poor,
 Or *blockhead porter* spurns them from your door.
 Ah, what avails your splendor, or brocade?
 The costly *bodice* with rich *stones* inlaid?
 The *shuttle's* grand productions cannot save,
 No more than *leather jackets*, from the grave.
 When the last *bell* proclaims departed breath,
 The good shall only then find *hope* in death.
 Think then in earnest how this life you spend,
 That *learning*, peace, and *concord* may await your end.

5. *A general Answer to the Enigmas and Charades. By the Revd. J. Shackleton, Thornton.*

Hail, publishers, may success attend
 On No. 2, for which I hope to send.
 Your correspondents favour the design;
 With enigmatic lays, if dark, yet fine.

The bell by Hindson, death by Wilfon sung ;	E. 1, 2
Campbell and wisdom are all neatly done.	Ch. 4, Pr.
Add Rofs, on leather, sweet as woodlark's strains,	E. 2, Ch. 10
With Taylor's waggon, Bodice wrote with pain.	E. 9, Ch. 2
Smart Tate on porter, cockboat also writes,	E. 7, Ch. 9
Whilst Kemp on time, blockhead, wine vaults indites.	E. 11, Ch. 5, 7
Bewly a loom or shuttle sure describes,	E. 10
As earnestly as earth the dew imbibes.	Ch. 8. E. 5, 6
On Stone, and Brighthelmstone, let Humber sing ;	E. 3, Ch. 1
Yet Day, like Verles shine as an ear ring ;	E. 8. Ch. 6
If of the rest I yet should have some doubt,	
Perhaps a bedstead may help me out.	Ch. 3

6. The Charades answered by Mr. J Hindson, Lincoln.

Brighthelmstone is a famous town ;	1
Bodices are old fashioned grown ;	2
Bedsteads often are desired ;	3
Campbell's genius is admired ;	4
Blockheads mostly are despised ;	5
Ear rings by the fair are prized ;	6
Wine vaults are the drunkard's glee ;	7
In earnest every soul should be ;	8
A cock-boat oft the sailor plies ;	9
The woodlark's echo charms the skies	10

Other Answers were also given by Messrs. Aristarchus, Allison, Adams, Bainbridge, Burton, Benson, Cicero, Dent, Denby, Fergason, Forster, Griffith, Harrison, Kempster, M^r Arthur, Orpheus, Robinson, Thoubren, Watson, and Young.

ANSWERS TO THE QUERIES.

1st Query, by Mr. Benjamin Kemp.

The joining or shaking of hands amongst friends, may be traced through many ages, and is a very ancient custom.—The first time we find it recorded in scripture, is at the meeting and ratification of friendship between Jehu and Jehonadab. See 2d Kings, Chap. X.

Answers are also given by Messrs. Aristarchus, J. Griffith, R. Humber, T. Longstaff, Orpheus, Wm. Robinson, Wm. Watson, and R. Young.

2d Query, answered by Mr. J. Hindson.

The meaning of the prophet is, that God will be served in sincerity. "He that killeth an ox" (without the disposition described in the latter end of the preceding verse;) "in order to sacrifice him, will be as great a crime in the sight of God, as if he slew a man: and he that sacrificeth a lamb, as if he had cut off a dog's neck, and offered him in lieu of the lamb." Now, a dog was held in great abhorrence by the Jews, insomuch, that even the price of a dog was forbidden to be brought into the house of the Lord. See Deut. 23 ch. verse 18.

The same answered by Mr. B. Kemp.

Isaiah ch. 66, verse 3d. "He that killeth an ox, is as if he slew a man." This is spoken of the Jews, because they thought themselves holy by offering their sacrifices, and in the mean time had neither faith nor repentance; therefore, God sheweth them in this verse, that he doth no less detest those ceremonies, than he doth the sacrifices of the heathens, who offered men, dogs, and swine, to their idols, which things were expressly forbidden in the law of the Jews.

Answers are also given by Messrs. Humber, Aristarchus, J. Griffith, T. Longstaff, Orpheus, C. Spence, Wm. Robinson, Wm. Watson, and R. Young.

3d Query, by Mr. B. Kemp.

The taste may be impaired or lost in sickness, from various disempers of the blood, as in fevers, &c. or from ulcers in the tongue and mouth, which destroy the sensation of the nerve *Papilla*.

The same answered by the Proposer.

It is owing (in my opinion) to some vicious humour, that choaks up the pores through which the savory particles of the food ought to pass; or there may (during the sickness) have happened some alteration in the organ by physic, &c. which is not then returned to its natural state.

Answers were also given by Messrs. Aristarchus, J. Griffith, T. Longstaff, J. M^r Arthur, Orpheus, J. Sarrat, Wm. Robinson, and Wm. Watson.

4th Query, by Mr. B. Kemp.

Surnames originated from the lands or possessions of either a person or his ancestors. They were used in England before the conquest, and long before they were used in Scotland, whither the English carried that custom. For when Margaret, Queen to Malcolm Canmor, King of the Scots, with her brother Edgar Atheling, fled into Scotland from William the Conqueror, many of the English who came with them and got lands, had their proper surname, such as Moubray, Lovel, Lisle, &c. using the particle *de* or *of* before them, which makes it probable they took them from the lands they or their ancestors possessed. About the year 800, the great men began to call their lands by their own names, but the ordinary distinctions were personal, and did not descend to succeeding generations, but either the name of the father, as John the son of William, &c. or the name of the office, as steward, &c. or accidental notes from complexion, stature, &c. as black, white, long, short, &c. or of their trade, as tailor, weaver, &c. The Lowlanders added son after the father's name, as Davidson, Robertson, Stevenson, &c.

The same answered by the Proposer.

* William the Conqueror told his officers and soldiers, who had come over with him, he thought the best way to have their names transmitted to pos-

* *Henderson's Life of the Conqueror.*

terity, would be, that every one of their sons should subjoin the name of the place from whence their fathers came to their own; and from this source did the institution of surnames arise. In the same manner, the designation of places took their beginning from the founder, as Rochester, from Roff, the river on which it stands; Maidstone, from Medway, &c. &c

Before the arrival of the Normans, men were usually named from their condition and properties, as Goddard, the Saxon word for good advice; and a woman was called from some quality of her body, as Swanshalfe, from the whiteness of her neck, &c. but after that period men began to be known by their dwellings, and to have an appellation from the possessions they enjoyed; the names, as at that time, of Thomas, John, &c. were introduced with others scriptural; and now in use among us. Such as had lands assigned them, were called from these; thus, if Thomas had got the township of Norton or Sutton, he was thenceforth called Thomas of Norton, or

of Sutton, &c. and in process of time was called Thomas Norton, or Thomas Sutton; others again preferred the name of the places in Normandy and Britany, whence they arrived; thus if a man came from a village called Vernon, Montague, &c. he transmitted to his posterity the surname of Vernon, or Montague, to be put after their Christian names so long as any of them should remain*.

Answers were also given by Messrs. Aristarchus, J. Griffith, J. Hindson, J. Hays, J. M^r Arthur, Orpheus, W. Watson, and T. Young.

NEW ENIGMAS.

1st Enigma, (13) by Mr. R. Humber, Brighton.

O, deign, celestial maid, to guide my quill,
And thy poetic strains on me distil;
Come, aid t'write, in easy flowing numbers,
Properties so rare—my partial wonders.
My primitive on yonder verdant plain,
Where warblers echo forth their dulcet strain;
And when that due consistency is giv'n,
I aid the irradiating rays of Heav'n:
Thus when an intimate is passing by,
To hail the friend, behold me cast up high.
My origin's from Indies' wide domain,
Unwrought, I'm borne across the ruffling main;
Prepar'd by artists for the cap-a-pee,
And martial warriors oft parade in me;

* See Lambert's Perambulation. P. 323.

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Anon I twine the tender infant's breast,
 And *Maria* when in Sunday's vest;
 Miss, deck'd in me, attends the ball and play,
 On some dear gala, or grand festive day:
 Oftimes I'm white, again a different hue,
 Sometimes am purple, yellow, black, and blue.
 I'm long, I'm short, of various sizes,
 And bought and sold at different prices;
 Ah! to my owner 'tis a sad vexation,
 That I endure the burden of taxation.
 Come, learned gents, and rend this slender veil,
 And tell the world the purport of my tale.

2d Enigma, (14) by Mr. J. Hindson, Lincoln.

In sultry Africa's domains
 Happy I liv'd upon the plains,
 Amongst the rural flock and herd,
 Till man, by curst av'rice led;
 He took me from my native home,
 Upon the raging seas to roam;
 And there, alas! with numbers more,
 Are sent to the West Indian shore,
 Bound and confin'd with locks and chains,
 We rend the air with our complaints:
 When there we come, I'm quickly sold,
 Or else exchang'd, ah! curst gold;
 Here all my woes are made complete,
 With cruel usage they me treat;
 And if I murmur or complain,
 I'm whipt, and flogg'd, and rack'd with pain:
 Hard is my lot; from day to day,
 I'm forc'd to work, and forc'd t' obey;
 Come, death, release me from the yoke.
 Cheerful I'll yield unto thy stroke;
 Back to my country I shall go,
 And bid farewell to grief and woe:
 My native home again shall see,
 Come, death, and come, sweet liberty.

3d Enigma, (15) by Mr. B. Kemp, Farnsfield.

When new born spring bids wint'ry scenes adieu,
 And opening tillage clothe the groves anew;
 While gentle showers invigorate the fertile earth,
 Gay nature smiles, and I receive my birth;
 Which glad event my mother loud proclaims
 In pleasing songs, that glad the village dames.
 Since by the housewife I am bought and sold,
 And fable says, I once was cloth'd in gold;
 My size is various, different is my hue;
 Am white, or speckled, sometimes green or blue;

Borne up aloft, 'twixt earth and heaven I'm found;
 Who soars the highest, leaves me on the ground.
 In search of me what numbers daily rove,
 And ruminat the woodland copse and grove:
 If me they find, they bear their prize away,
 Pleas'd with their gain, ne'er grudge the toilsome day;
 Yet so concealed by a parent's care,
 I oft elude the subtle lurcher's snare:
 For should I 'scape, and from my bonds get free,
 A numerous train originate from me,
 Which constitute the links in nature's chain,
 From smallest insects to leviathan:
 Yet oft I'm caught, and in embryo spoil'd,
 My armour broke, and all my riches foil'd;
 Priz'd when I'm good, if bad, a proverb made
 To please a rabble, or a knave degrade.

4th Enigma (16) by Mr. Thomas Nield.

The roaring winds, the lightning's flash,
 The falling rains, the thunder's crash,
 Affright the weak untutor'd youth,
 Devoid of knowledge—sacred truth:
 But I, the grand perfectioner of all,
 Wear off the dread, or make it seem but small.
 When Emma, frail, with direful art,
 Attracted, repelled, kind Edwin's heart,
 Until she found it was her own,
 Then left him, (faithful youth) to moan:
 Alas, now Edwin must rely on me
 To ease his breast, and set his troubles free.
 When death, with all his terrors, strike
 A favourite son—the parent like
 With mournful look bends o'er his bed,
 Until his breath's entirely fled;
 To me they then for soothing balm apply,
 I grant their boon, their sorrow's turn to joy.
 Now shew the world, pray, what and who am I. }

5th Enigma, (17) by Mr. Carwithen, Ripley.

When Adam from the earth his head uprear'd,
 The great omnipotent work rever'd,
 Surveyed the mould and the ethereal sky;
 Birds, insects, beasts, beneath their canopy.
 All those he saw were different from his frame,
 Not one could the omniscient work proclaim,
 Except himself, yet each had a mate;
 Pondering, thus he did expatiate:
 Why am I here alone on this fair plain,
 Without a partner to fill up my train?

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Am I more despicable than the brute,
 Or should I with a fellow mate dispute?
 Methinks I could support society;
 Do thou, great Donor, form the social tie.
 Thus did he mourn till he beheld a bride;
 Then for a while this pair were satisfied;
 Till trade encreased, and profits did accrue,
 And fashionable speculation grew:
 Then were they few that did require my aid,
 Except at sea, or some laborious trade;
 For who could wish, where property's at stake,
 Another should of their earn'd gains partake?
 But speculation is a choice parterre,
 Thousands at me, may of her bounty share;
 And should she fail to lend her powerful charm,
 If I am there I half her dart disarm;
 Then blame me not if sometimes I intrude,
 And break abruptly on your solitude.
 My motive's not always to give offence,
 Tho' by the fair accused of impudence:
 But these are trifles, mere coquetish toys,
 And used by fastidious girls and boys,
 Who pride themselves on their own consequence,
 But I am highly priz'd by men of sense,
 Who friendly do their thoughts communicate,
 No fordid gain their minds contaminate;
 To all alike, those freely give access,
 Their useful thoughts as freely do express;
 Then censure not my fascinating name,
 For none without me mount on wings of fame.

Prize Enigma, by Mr. Nield, Master of Overton Boarding School.

[Whoever answers this by the first of March, has a chance, by lot, to win twelve Companions.]

Some sing of warriors rare, and giants great,
 Of witch and wizard some will tales relate;
 But I a nobler theme have got in view,
 And greater wonders will present to you;
 Let me come forth, I'll clear the way;
 Without command, or order, all obey;
 In rank and file along my sides they pass,
 Least I should touch them, seem in dire distress;
 For, should I give them e'er so small a pat,
 I straight should leave them black as any hat.
 At my approach the little children run,
 From puny Mifs, to the lusty well grown son.
 The ways are black that I do mostly prize,
 While step by step I then tow'rds heaven rise;

And, like the doctors, purge the way I take,
 When I but do my curious weapon shake:
 When I'm exalted far above each friend,
 Like chanticleer I then my throat extend;
 And one shrill note's repeated in the air.
 The work is done, and soon I disappear:
 So let my name be seen another year.

NEW CHARADES.

1st Charade, by Mr. R. Humber, of Brighton.

My first doth stem the purling brook,
 Where weeping willows grow;
 My next a ruddy darling youth,
 With health upon his brow:
 My whole a fruit of blooming hue,
 Near Partlet's cottage door,
 The truant boy oft scales the fence,
 To rob the rustic poor.

2d Charade, by Mr. T. Nield.

In my second, how glorious my first does appear,
 On my whole the bells merrily ring;
 It was made for a blessing for us mortals here,
 And the praise of its maker we sing.

3d Charade, by Mr. J. Hindson, Lincoln.

Of my first I hope you have two,
 My next on the head may be found;
 My whole will disclose to your view,
 A foreign and large trading town.

4th Charade, by Mr. J. J. Thompson, of Brighton.

My first sustains most human kind,
 My second's a strong fence and show;
 My whole a county gay you'll find,
 Where riches in abundance flow.

5th Charade, by Mr. W. Robinson, London.

Behold my first the miser's care,
 When contracts he doth make;
 My next your health is sure t' impair,
 If death be not your fate.
 O direful whole! may I be ever free
 From all thy chains, and be at liberty.

6th Charade, by Mr. George Ross, Rotherham.

From climes remote my first is brought,
An import to this Isle;
My tuneful next, to please each sex,
Will oft an hour beguile:
My whole the aged take, you'll find,
For consolation to the mind.

7th Charade, by Aristarchus, Hamsterly, Durham.

To three fifths of a liquid that's very much us'd,
Add the half of a poem by the lover produc'd;
These rightly combin'd, will appear to your view
The name of my friend, who is sincere and true.

NEW REBUSSES.

1st Rebus, by Mr. J. Hindson, Lincoln.

Connect a preposition much in use,
To what the merry fiddler can produce;
Propitious whole, thy influence kindly shed,
And show'r thy general blessings on my head.

2d Rebus, by Mr. B. Kemp, Farnsfield.

Two thirds of the traveller's welcome retreat,
And the food which the serpent was destin'd to eat;
To these, if you please, add the third of a grain,
And a laudable virtue the whole will explain.

3d Rebus, by the Rev. J. Shackleton, Thornton.

Of neither enigma nor charade, four fifths please to take,
Then call in the tailor a new suit to make,
What he puts on your coat sleeve, one half thereof's my next;
If you meet with my whole perhaps you'll be vex't.

bton.

4th Rebus, by Mr. R. Humber, Brighton.

One third of an insect, with a sting in his train,
And part of a fish that swims in the main,
With the head of a nut, if rightly combin'd,
The name of a damsel that's virtuous you'll find;
Who is lovely and chaste, beauteous and fair,
And breathes in fam'd Brighton's salubrious air.

5th Rebus, by Mr. J. Hindson,

One third of what mostly is used at tea;
Ten hundred and two eights of a measure;
When rightly connected, you'll then plainly see,
A genius whose lays give me pleasure.

6th Rebus, by Mr. J. Griffith, London.

By the side of my first the shepherd sat down,
 Whilst my second was sweetly thrill'd o'er;
 Wanting not my whole to give him content,
 Though he had no riches in store.

7th Rebus, by Mr. T. Nield, of Overton.

To four sixths of the death that king Charles the first died,
 Add the whole of a small preposition;
 My friend they will name, who's a stranger to pride,
 Tho' of a funny and droll disposition.

Answers to the last Year's Charades and Rebusses

CHARADES.

- 1 BRIGHTHELMSTONE
- 2 BODICE
- 3 BEDSTEAD
- 4 CAMPBELL
- 5 BLOCKHEAD
- 6 EAR-RING
- 7 WINE VAULT
- 8 EARNEST
- 9 COCK-BOAT
- 10 WOODLARK

REBUSESSES.

- 1 CANARY
- 2 LINNET
- 3 HUMBER
- 4 CONCORD
- 5 LAMB
- 6 HOPE
- 7 MUSIC
- 8 ORK
- 9 LEARNING
- 10 JARVIS

NEW QUERIES.

1st Query (5), by Mr. R. Humber, of Brighton.

Matthew, 2nd chap. verse the 23rd. And he came, and dwelt in a city, called Nazareth; that it might be fulfilled which was spoken by the prophets, he shall be called a Nazarene. Query. Which of the prophets? the word Nazareth not being known in the Old Testament; or how is it to be understood?

2d Query (6), by Mr. T. Hewitt, Teacher of the Mathematics, Bunhill Row, London.

Ye learned in numbers, how will you contrive,
 To prove that Nothing is equal to Five?

3d Query (7), by Mr. J. Hindson, Lincoln.

Cellars appear to be much warmer in winter than in summer. Query, the reason?

4th Query (8), by Mr. B. Kemp, Farnsfield.

2nd Kings, chap. 2, verses 23, 24. Wherein consisted the propriety of Elisha's conduct in cursing the children of Beth-el, seeing that manner of cursing was forbidden in the Law of Moses?

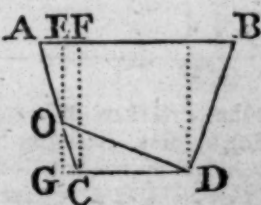
5th Query (9), by Mr. William Robinson, London.

What is the meaning of our Saviour in the 60th verse of the 9th chapter of St. Luke's Gospel, where he says, "Let the dead bury their Dead?"

ANSWERS TO THE QUESTIONS PROPOSED LAST YEAR.

1st Question answered by Mr. Thomas Coulberd, Student in Mr. Jackson's School, Frosterly, Durham.

Let the lines be drawn as per figure, and put x the sine of the angle $AOE = GOC$; then as rad : $AO :: \sin \angle AOE : AE = 30x$, and $AF - AE = GC = 10 - 30x$; also by the similarity of the triangles GOC , and AOE , as $AE : AO :: GC : CO = \frac{10 - 30x}{x}$ and

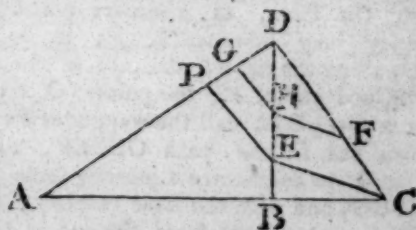


by a common theorem, $CD^2 + CO^2 + 2 CD \times CG = OD^2$, or $48x^3 + 55x^2 + 6x = 1$, from which $x = .0887935$, the depth $CF = 112, 176$ inches, and the content $25 + 1.02$ ale gallons.

Other answers were also given by Messrs. Aristarchus, T. Allison, W. Armstrong, T. Baron, S. Beacroft, J. Crawford, R. Dean, S. Dent, R. Furby, A. Griffith, J. Griffith, T. Hewitt, H. Harvey, R. James, G. Langford, A. Murdock, R. Pearson, G. Read, W. Robinson, J. Surtees, J. Thoubren, R. Thompson, and R. Young.

2nd Question answered by Mr. John Johnson, of Birmingham.

Construct the right angled triangle BDC , having BD the given $\perp$ and DC the given leg, produce BC to meet AD drawn $\perp$ to DC , in A ; take any point H in BD , from which draw HG perpendicular to AD , and from H to DC , apply $HF = HG$, draw CE parallel to FH , and E is the point required.



Demonstr. for draw EP perpendicular to AD , then since $GH = HF$, and HF parallel to CE , and GH parallel to PE , $PE = EB$.

whence $v = \sqrt{r \times r - \frac{2a}{t}} = \sqrt{1239400} = 3520$ yards, or 2 miles per hour the required rate of travelling.

The same answered by Mr. Coultherd.

Let x be the number of miles the team travelled per hour, and C and v the points from which they began to move; then when the man had gone 2 yards, or was at B ,

the team would be at n , or $\frac{8+2x}{4}$ yards before him: and,

as $4-x$; 4 ; $\frac{104+2x}{4}$; $\frac{104+2x}{4-x}$ the distance between

B and M , to which add $CB = 2$ yards, the sum $\frac{112}{4-x}$ will

be the number of yards from C to M . And by a similar

process, the distance from M to $O = \frac{112}{4+x}$ hence $\frac{224}{3}$

$= v \frac{112}{4+x} + \frac{112}{4-x}$ and $x = \sqrt{\frac{896}{224}} = 2$ the ratio required.



Other Answers were given by Messrs. Aristarchus, Adams, Allison, Armstrong, Baron, Bearcroft, Crawford, Dean, Dent, Gale, Hewitt, Harvy, Leach, Langford, Moody, Robinson, Read, Surtees, Thoubren, Tale, and Young.

Question 4 answered by Mr. T. Coultherd.

It is well known, that two chords, having the same tension, and whose lengths are to one another as 2 to 3, when made to vibrate, will produce a perfect fifth. And as the solidity of similar figures, and consequently their weights, are in the triplicate ratio of their homologous sides; therefore as $3^3 : 2^3 :: 10000 : 2962'96$ the weight required.

Other Answers were also given by Messrs. Aristarchus, Andrews, Bell, Barns, Gale, Griffith, Hewitt, Robinson, and Surtees.

Question 5 answered by Mr. T. Coultherd.

Let the given line $AB = a$, tang. $\angle A$

$= p$, and $x = AC$; then as rad. : $a :: \text{tang.}$

$\angle A : DC = px$, and $CB^2 + CD^2 = BD^2$

$= p^2 x^2 + a^2 - 2ax + x^2$; also $CB \times$

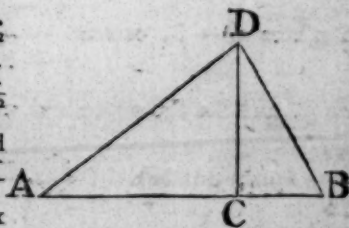
$BD^2 \times CD^3 = p^3 x^3 \times a - x \times p^2 x^2 + a^2$

$- 2ax + x^2 a$ maximum, which fluxed

and reduced, is $= 1 + p^2 \times 6x^3 - 5ap -$

$15a \times x^2 + 12a^2 x = 3a^3$; from which x

is easily determined.



Other answers were also given by Messrs. Aristarchus, Adams, Allison, Armstrong, Baron, Crawford, Crowie, Dent, Dean, Gale, Griffith, Harvy, Hill, Kirby, Langford, Lisle, Murdock, Orpheus, Oliver,

Pearson, Robinson, Reed, Surtees, Simson, Thoubren, Tompkins, Young, and the Proposer.

Question 6 answered by Mr. T. Coultherd.

Let the surface of the water np , by the immersion of the cone, be raised to mo , and put $rs = x$; then, as $EF - GH: Gv :: EF - rs; qr = 30 - 3x$, and the solidity of the frustrum $EFrs = \frac{1000}{4}$

$$\frac{34107x}{4} \times n; \text{ also } \overline{rq} - ru \times CD^2 \times n = \frac{35594}{4}$$

from which $x = 8.23703$; consequently $26 - 3x = 1.2889$ inches, the height required.

Other answers were also given by Aristarchus, Allison, Adams, Baron, Crawford, Dent, Furby, Gale, Griffith, Hewitt, Langford, Lisle, Moody, Orpheus, Robinson, Reed, Surtees, Thoubren, Yates, and Young.

Question 7 answered by Mr. Surtees, Minor, Alstone, Cumberland.

Let x^2, nx^2, n^2x^2 , be the three numbers; then per question $\overline{n^2 + n + 1} \times x^2 = \square$, or $n^2 + n + 1 = \square$; put $ne - 1 =$ its side, then $n^2e^2 - 2ne + 1 = n^2 + n + 1$, hence $n = \frac{2e+1}{e^2-1}$, and the three

numbers will be $x^2, \frac{2e+1}{e^2-1} \times x^2$, and $\frac{2e+1}{e^2-1}^2 \times x^2$, where x may be taken any number at pleasure, and e greater than one.

The same answered by James Gale, Esq. Edgeware Road, London.

Suppose the numbers to be x^2, x , and 1 ; then $x^2 + x + 1$ must be a square number, the root of which put $= x - n$: then $x^2 + x + 1 = x^2 - 2nx + n^2$, or $x + 1 = -2nx + n^2$, and by transposition $x + 2nx = n^2 - 1$, or $x = \frac{n^2 - 1}{1 + 2n}$. If n be taken $= 2$, then x will be

$= \frac{3}{5}$, and the three numbers will be $1, \frac{3}{5}$, and $\frac{9}{25}$ the sum of which is $\frac{49}{25}$ — a square number.

The 3 numbers are likewise

$$\left. \begin{array}{l} 1 + p + p^2 \\ p(1 + p + p^2) \\ p^2(1 + p + p^2) \end{array} \right\} \text{ When } p \text{ any integer} \\ \begin{array}{l} 42 \mid 7 \cdot 14 \cdot 28 \\ 43 \mid 10 \cdot 39 \cdot 107 \end{array}$$

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The same answered by Mr. James Thoubren, Student in Mr. Rutherford's School, Lancaster.

Let 1, x , and x^2 represent the required numbers, then $1 + x + x^2$ must be a square number, the side of the square of which put $= m - x$, then $1 + x + x^2 = m^2 - 2mx + x^2$ or $1 + x = m^2 - 2mx$, whence $x = \frac{m^2 - 1}{2m + 1}$ and the three numbers are $1, \frac{m^2 - 1}{2m + 1}, \frac{m^2 - 1}{2m + 1}^2$ respectively, and by taking $m = 3$, the numbers will be $1, \frac{8}{7}, \frac{64}{49}$.

The same answered by Mr. T. Coultherd.

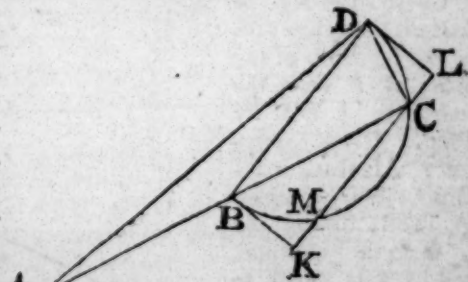
Let x^2 be the first number, n their ratio, and each of the other numbers will be represented by nx^2 , and n^2x^2 ; then per quest.; $n^2 + n + 1 \times x^2$, or $n^2 + n + 1$, must be a square number; assume $n + a$ for its root, and $n^2 + n + 1 = n^2 - 2an + a^2$ and $n = \frac{a^2 - 1}{2a + 1}$ therefore the three numbers will be $x^2, \frac{a^2 - 1}{2a + 1} \times x^2$, and $\frac{a^2 - 1}{2a + 1}^2 \times x^2$ where x and a may be taken any numbers at pleasure, provided a be greater than one.

Other answers were also given by Messrs. Aristarchus, Adams, Baron, Dent, Edwards, Griffith, Hewitt, Johnson, Langford, Moody, Orpheus, Robinson, Reed, Simpson, and the Proposer.

8th Question answered by Mr. J. Edwards, Jun. Hoxton.

Let BD be the given besection line, upon which describe a segment of a circle, to contain the given angle; and on BD constitute the rectangle BDLK $=$ to the given area; join B & C, the point of intersection of the circular segment with KL; and in CB produced, take AB $=$ BC; join DA and DC, and ADC is the Δ A required. For AB $=$ BC by constr. and by the 38 and 41, Euc. I, the Δ ADC $=$ $\square$ DBKL $=$ given area by construction.

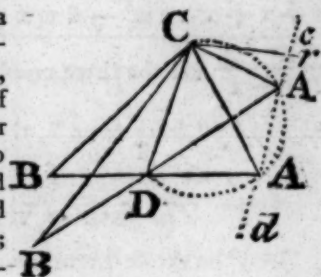
When the circle cuts KL in two points, MC, as in the present case, the question admits of two solutions; and when it only touches it, the



area is a maximum, and when it neither cuts nor touches it, the prob. is impossible.

The same answered by Mr. John Surtees, Minor, Alstone, Cumberland.

Confr. Let M be equal to the side of a square, expressing the given area of the triangle. On CD , the given bisecting line, describe the segment of a circle capable of containing the given angle; and at C erect $Cr \perp DC$, and equal to a third proportional to DC and M ; through $r \parallel$ to DC , draw d Arc, cutting the circle in A ; join AC and AD , and in AD produced, take $DB = AD$; join CB , and ABC will be the triangle required.

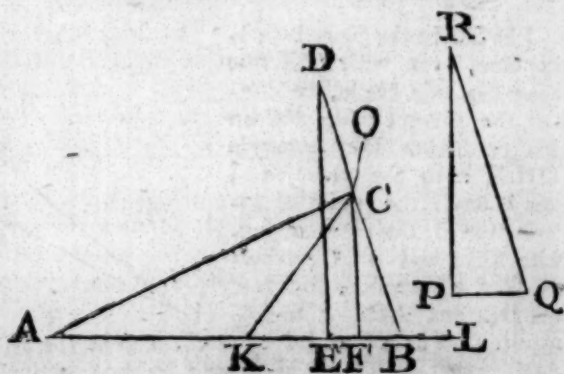


For the $\triangle BCD = \triangle DCA$ (Euc. 6. 1.) $= \frac{1}{2} DC \times Cr$, whence the $\triangle BCD + \triangle DCA = DC \times Cr = M^2$ equal to the given area by constr. and $\parallel$ lines; and the angle CAB and DC are $=$ to the given angle and bisecting line by construction.

Other answers were also given by Messrs. Aristarchus, Adams, Baron, Coultherd, Dent, Gale, Griffith, Hewitt, Johnson, Murdock, Orpheus, Robinson, Thoubren, and the Proposer.

9th Question answered by Mr. Joseph Edwards, Jun. the Proposer.

Confr. Let KF be half of the given diff. of the segments of the base; at F erect the indefinite $\perp FO$, by Prop. 62 of Simpson's Algebra, construct the right-angled $\triangle PQR$, having the $\angle Q =$ the given one, and its area equal to half the given area; in KF produced, take FB , so that $FB \times BK$

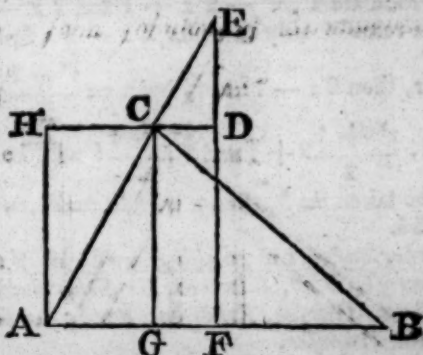


$= PQ^2$, draw BC , making the $\angle KBC =$ the given $\angle Q$; take $KA = KB$, and from the point C , where BC cuts the indefinite $\perp FO$, draw CA , and ABC will be the triangle required.

For draw KC , and in KB take $BE = PQ$, and at E erect $ED \perp KB$, meeting BC produced in D . Now it is evident that the right angled Δ s EDB, PQR , are sim. and equal, and, each = to half the given area by constr. But the Δ s FBC, EBD , are also sim. therefore, $\Delta FBC : \Delta EBD :: FB^2 : EB^2 (= KB \times BF \text{ by constr.}) :: FB : KB :: \frac{1}{2} FB \times FC : \frac{1}{2} KB \times FC$ (Euc. 5. 15) $:: \Delta FBC : \Delta KBC$, whence the antecedents FBC and FBC being the same, the consequents EBD, KBC will be equal i. e. the $\Delta EBD = KBC = PRQ =$ to half the given area, (by constr.) $= \frac{1}{2} ACB$ (Enc. 5. 1.)

The same answered by Mr. John Johnson, of Birmingham.

Let M^2 be equal to the given area, $DC =$ half the given difference of the segments of the base; make the $\angle DCE =$ the given angle at the base, and draw DEF and EG perpend. to CD ; continue DC to H , so that rect-angle CHD , (Simpson's Geo. 9. 6.) may have to the M^2 the ratio of $CD : DE$, draw HA , perpendicular to HC to meet EC produced in A draw $AB \parallel$ to CD meeting ED produced in F , make $FB = AF$, and join CB , and ACB is the triangle required.



Demonstr. Because $AF = FB$, therefore $GB - AG = 2 GF = 2 CD =$ the given difference of the segments of the base by constr. Moreover, by reason of $\parallel$ lines the $\angle BAC = DCE =$ the given one; also by sim. Δ s $AG : GC :: CD : DE$; but by constr. $CHD (AG \times AF) : M^2 :: CD : DE$, therefore by equality $AG \times AF : M^2 :: AG : GC :: AG \times AF : GC \times AF$ (Euc. 5. 15.) whence the antecedents being the same, the consequents must be equal, i. e. $GC \times AF = M^2$ equal to the given area by construction.

Other answers were also given by Messrs. Aristarchus, Adams, Baron, Coultherd, Dent, Griffith, Orpheus, Robinson, Surtees, Thoubren, and Young.

Question 10 answered by Mr. John Johnson, of Birmingham.

Let x, y and z be the numbers sought, then $x - y, x - z$, and $y - z$, must be square numbers, which put respectively equal to $16v^2, 25v^2$, and $9v^2$, then by transposition $x = 16v^2 + y, x = 25v^2 + z$ and $y = 9v^2 + z$; whence v and z may be assumed at pleasure. Assume $v = 1$ and $z = 1$, then $y = 10$, and $x = 26$, whence $x - y = 26 - 10 = 16 = 4^2, x - z = 26 - 1 = 25 = 5^2$, and $y - z = 10 - 1 = 9 = 3^2$.

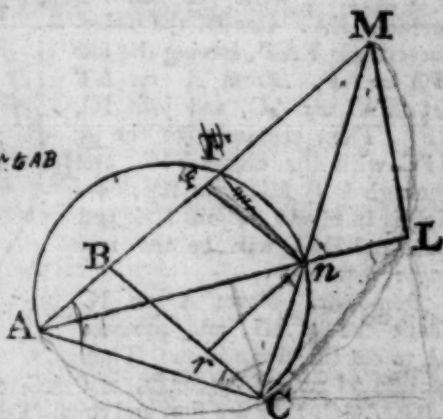
The same answered by Thomas Coultherd.

Let $5x^2, x^2$, and z be the three numbers; then $5x^2 - x^2$ is evidently a square; but to make $z - x^2$ a square also, assume $bb = z - x^2$, and $z = bb + x^2$ will be a square. It now remains to make $5x^2 - z$ a square, for

32. 1.) *therefo.* $\angle ABC - CAB = \angle DEC = DEA =$ given angle (by *constr.*), and because the $\triangle DEC$ and ECB are equal, the $\triangle DEA$ is equal to the diff. of the areas of the $\triangle ACE$, $ECB = AD \times \frac{1}{2} A = M^2$ equal to the given magnitude by *constr.*

The same answered by Mr. Joseph Edwards, Junr. the Proposer.

Constr. In the indefinite line AM take $AB =$ to half the given diff. of the sides, make the $\angle BAC =$ the complement of half the given diff. of the $\angle$ s at the base, and from the given point B draw the $\perp BC$ meeting AC in C ; upon AC describe a segment of circle to contain $\angle = \angle BAC$, and in BC take Br , so that $Br \times BA$ may be $=$ the given diff. of the areas; draw $rn \parallel AM$, meeting the circle in n , through which point and C draw CnM , meeting AM in M , and from M draw ML , making the $\angle CML = \angle CMA$. Lastly, from A through n draw AnL , meeting ML in L , and AML will be the triangle required.



Demonstr. Draw $nF \parallel CB$. By *constr.* the $\angle BAC = \angle AnC = \angle L n M =$ to the complement of half the given angle by *constr.* therefore the $\triangle s n ML$, CMA are *sim.* and consequently the $\angle n LM = \angle ACM$, a circle will therefore pass through the points $ACLM$; moreover MC bisects the $\angle AML$, therefore by *prop. 13 Simpson's Trig.* BA will be $=$ half the diff. of the sides AM , $ML =$ half the given diff. by *constr.* and because MC bisects the $\angle AML$, $\frac{1}{2} n F \times AM + \frac{1}{2} n F \times ML =$ the area of the $\triangle AML$, and *conseq.* $n F \times AB =$ to the difference of the areas of the $\triangle s AM n$, $n ML$, equal to $BA \times Br$, equal to the given diff. by $\parallel$ lines and *constr.*

Other answers were also given by Messrs. Aristarchus, Baron, Coulter, Johnson, Orpheus, Robinson, Thoubren, and Young.

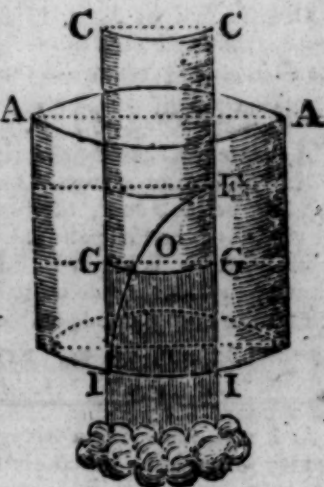
$$\begin{aligned} \angle n C &= \angle M A L + \angle A n L \\ \angle n C &= \angle M A L + \angle A n L \\ \angle n C &= \angle M A L + \angle A n L \\ \angle n C &= \angle M A L + \angle A n L \end{aligned}$$

NAB will be the Δ required. For the prob. is evidently impossible, when the circle does not touch KL; and the $\angle$ in the circular segment is greater than any other formed by lines drawn from AB, to meet in any other point in KL (Ecu. 1. 32.)

Other answers were also given by Messrs. Aristarchus, Baron, Coulter, Moody, Orpheus, Robinson, Surtees, and Young.

The Prize Question answered by Mr. Surtees, Minor, Alstone, Cumberland.

Let b = the area of the cylinder, a = its altitude, x = EI any variable altitude, d = II = GG, $s = 16\frac{1}{2}$ feet, and suppose the piece CC, GG, to be drawn up n feet per second; then tn (= GI) = the height drawn up in any time t : also let EOII be a parabola—Then (per Emerson's Mechanics, corol. 6th. prop. 97) $dt \sqrt{2sx}$ = the quantity running out (per second) of a hole = GGII at x depth; and by corol. 2, prop. 98, same book. As the area GGII : the area GOII :: $dt \sqrt{2sx}$: $\frac{2}{3} dx \sqrt{2sx} - \frac{2}{3} \times x - tn \times \sqrt{x - tn}$ $\times 2s$ = the true quantity running out of the hole IGGI per second, which being divided by b , will give the velocity of the descending surface. Now by the laws of



$$\text{motion, } \dot{t} = \frac{-\dot{x}}{v} = \frac{3b}{2d\sqrt{2s}} \times$$

$$\frac{-\dot{x}}{x\sqrt{x - x - tn} \times \sqrt{x - tn}}, \text{ the fluent of which being found, and } x$$

taken = tn it will give t = the time in which the surface of the water will descend to the top of the hole.

Again. Let x = any variable height less than tn ; then (by Cor. 1. same prop.) $\frac{2}{3} dx \sqrt{2sx}$ = the true quantity running out per second after the surface of the water is below the top of the hole, and divided by b , gives the velocity of the descending surface. By the laws of motion. $\dot{T}$

$$= \frac{-\dot{x}}{v} = \frac{3b}{2d\sqrt{2s}} \times -\dot{x}x, -\frac{1}{2} \text{ hence } T = \frac{3b}{\sqrt{2sx}}$$

$\frac{3b}{d\sqrt{2s}tn}$, and when $x = 0$, T is infinite, so of consequence $T + t$ (= the time of a compleat exhaustion) is also infinite.

NEW QUESTIONS,

To be Answered in the Next COMPANION.

1st Question (14) by Mr. T. Hewitt, of Bunhill-Row.

ON the first of June, 1798, the sun's true amplitude from the east, towards the north, and his declination in one sum, was found to be equal to the co-latitude; where did this happen?

2nd Question, (15) By Mr. Thomas Coultherd, Frosterly.

A gentleman bought the right of enclosing two fields; the one is to be in the form of a circle, and the other in that of an Ellipse, whose two axis are as 3 to 2, the sum of the greater axe, and the diameter of the circle is to be 120 poles; also for every square pole, enclosed in the circular field, he is to pay 6 shillings, and for the same quantity in the other, 4 shillings. I desire to know the content of each field, supposing he hath the least ground possible for his money.

3d Question (16) By Master James Thoubren, at Mr. Rutherford's Academy, Lancaster, Durham.

As I was surveying a triangular piece of ground, I observed a road that lay through it, which divided its area into two equal parts, cutting the base, or longest side, at an angle of 40° , and passing through the vertical angle. Also the side opposite to the given angle is 7 chains, and the whole vertical angle $86^{\circ} 14'$. From hence it is required to find the area, the length of the said road, admitting it to be a straight line, and the other two sides of the triangle.

4th Question (17) by Mr. Thomas Coultherd.

In one of the front windows of the school room of Frosterly, which declines $16^{\circ} 30'$ from the south towards the west, I observed a pen 6 inches long, stuck up against the glass; the lower end at the distance of 4 inches from the west side of the window, and the other inclined towards the east side at an angle of 30° with the horizon. I desire to know what distance the upper end of the quill was from the extreme point of its shadow, upon the upright plane on the side of the window, which makes an angle with the glass of an 100° at 10 o'Clock on Lammas day, 1798, (true time) the latitude being $54^{\circ} 56'$.

5th Question (18) by Mr. William Hilton, Liverpool.

Given the base and vertical angle of a plain triangle to construct it;

when the rectangle under the sum of the sides, and the line bisecting the vertical angle, and terminating in the base, is of a given magnitude.

6th Question (19) by Mr. Joseph Edwards, Junior, Hoxton.

It is required to find a point p , in the diameter of a given semicircle, such that, if from it two right lines AP and PB be drawn to meet the circumference, the former, in a given point A , and the latter $\perp$ to the diameter of the given semicircle, and meeting the circumference in B , that the difference of their squares may be equal to a given rectangle, likewise to determine a point in the diameter, from which, if two right lines be drawn as above, the ratio of their squares may be given.

7th Question (20) by Mr. James Woolfsenden.

The square of half the difference of the sides, less the square of half the difference of the segments of the base, made by the line bisecting the vertical angle, is to the square of the segment of this line, included betwixt the base, and a perpendicular demitted thereon from either extremity of the base, as one side of the triangle is to the other. Required the demonstration.

8th Question (21) by Mr. John Fletcher, of Liverpool.

In a plane triangle are given the vertical angle and the sum of the including sides, to determine it when the rectangle under the base and the difference of the said sides is of a given magnitude.

N. B. The above Question is Question 174 in Mr. Whiting's Delights, and is there solved algebraically; but, as it admits of a neat geometrical construction, such a one is here required.

9th Question (22) by Mr. William Hilton.

Given the base and vertical angle of a plane triangle to construct it when n times the greater segment of the base, made by the perpendicular from the vertical angle added to m , times that perpendicular is a given quantity, or a maximum, n and m being given numbers.

10th question (23) by the same.

In a given segment of a circle, $ADCB$, described on any given line AB , it is required to inscribe the chord CD given in length, so that the rectangle of the perpendiculars demitted from C and D on AB (produced if necessary) may be of a given magnitude.

11th Question (24), by Mr. John Fletcher, of Liverpool.

Three points, A , B , and C , being given, it is required to find a fourth D , from which, if lines be drawn to the former three, $m \times AD^2$, $n \times BD^2$, and $p \times CD^2$, shall have given differences.

12th Question (25) By Mr. John Surtees, Alstone, Cumberland.

It was said that a Newcastle Keelman had undertaken, for a trifling

wager, to jump, or drop himself from the top of Wearmouth bridge, at Sunderland, into the river; and, of course, then to swim out again; and on the day fixed, a number of people assembled to see him perform, but were disappointed.

Now, supposing him to have dropt from the top of the paling on the center of the said bridge, which is at least 116 feet above the bed (or bottom) of the river. Query, With what velocity will he strike the bottom? and also, whether he would receive any injury by the stroke; admitting that he would meet with resistance, equal to that of a globe of the same density, and one foot diameter; depth of the water = 22 feet, and density of the air, water, and the globe, or Keelman, to be as the numbers $1\frac{1}{5}$, 1000, and 1030 respectively?

13th Question, (26) by Mr. Edward Bulman, Alstone, Cumberland.

Seeing an Exciseman's staff, in form of a cylinder, $\frac{3}{4}$ of an inch diameter, and 36 inches long, immersed in a vessel of beer at one end, and the other resting on the edge of the vessel, three inches above the liquor; I observed 13 inches along the staff's axis to be dry. Required the weight of the staff, a cubic inch of beer weighing 5949 oz. averdupois.

We have, at the desire of several of our ingenious correspondents, re-proposed the above question; the answers hitherto given not appearing satisfactory to them, as most of them are of an opinion, that some weight must rest on the edge of the vessel.

14th Question, (27) by Mr. George Sanderson, London.

It is required to describe the line that is the locus of this equation, $\frac{bx - ba}{a} = y$; and to find the absolute values of y , when $x = \frac{3a}{2}$; $\frac{a}{2}$, and $-\frac{a}{2}$ respectively.

The reason of re-proposing this simple question will be given with an answer in the next Companion.

15th Question (28), by Mr. Joseph Edwards, Sen. Hoxton.

A certain Gentleman's house is supplied with water from a neighbouring reservoir, by means of a straight slender horizontal pipe, d feet long. It is observable, that the velocity of the effluent water increases after the cock is opened. Query, the velocity of the water in the pipe after the cock has been opened any given time; the reservoir being constantly full, and its surface c feet above the pipe?

Prize Question, by Joseph Edwards, Sen.

[Whosoever answers it by the 1st of March, has a chance, by lot, to win twelve Companions.]

If a ball A , whose diameter is equal to c , move upon a smooth horizontal plane, with a velocity equal to d feet per second, and strike the quiescent

ball B, whose diameter is equal to 2 C: Required, their velocity after collision, supposing the direction of B in a vertical plane, passing through both their centers; and also, that both bodies are elastic, and of equal density.

A Discourse on the Locus for three and four Lines, celebrated among the ancient Geometers. By H. Pemberton, M.D.R.S. Lond. et R. A. Berol. S. In a Letter to the Reverend Thomas Birch, D.D. Secretary to the Royal Society.

SIR,

Dec. 15, 1763.

Read at R. S. MY worthy friend, and associate in my early studies, 15 Dec. 1763. the collector of the late Mr. Robins's mathematical tracts, thought it conducive to a more compleat vindication of the memory of his friend against an insinuation prejudicial to his candour, to make some mention of the course I took in my early mathematical pursuits, and how soon I became attached to the ancient manner of treating geometrical subjects. This gave occasion to my looking into some of my old papers, amongst which I found a discussion of the problem relating to the *locus ad tres & quatuor lineas*, celebrated among the ancients, which I then communicated to a friend or two, whose sentiments of those ancient sages were the same with mine. What I had drawn up on this subject is contained in the papers I herewith put into your hands, which if you shall think worthy of being laid before our honourable society, they are intirely at your disposal.

I am your most obedient servant,

H. PEMBERTON.

THE describing a conic section through the angles of a quadrilateral with two parallel sides, is so ready a means of assigning *loci* for the solution of solid problems, that it cannot be doubted; but this gave rise to the general problem concerning three and four lines mentioned by Apollonius, and described by Pappus; and it may be learnt from Sir Isaac Newton, who has considered the problem, how easily the most extensive form of it is reducible to the case, which I have supposed to give rise to it.

Sir Isaac Newton refers the general problem to this: any quadrilateral ABCD being proposed, to find the *locus* of the point P, whereby PRQ being drawn parallel to AC, and SPT parallel to AB, the ratio of the rectangle contained under QP, PR to that under TAB. XXIV. SP, PT shall be given; and this by pursuing the steps, Fig. 1. whereby he proves, that the point P will in every quadrilateral be in a conic section, may be readily reduced to the case of a quadrilateral with two sides parallel, after this manner. Draw Bf and DN.

D. 3.

parallel to AC, then find the point M in ND, that the rectangle under MDN be to that under ANB in the ratio given, and draw Cr Md:

Here Rr will be to AQ, or SP, as MD to AN, and Bt, or QP, to Tt as ND to NB whence the rectangle under Rr, QP will be to that under SP, Tt as that under MDN to that under ANB, that is, in the ratio given of the rectangle under RPQ to that under SPT. Therefore, by taking the sum of the antecedents and of the consequents, the rectangle under rPQ will be to that under SPt, that is, to the rectangle under AQB, in the quadrilateral ABCd, whose two sides AC, Bd, are parallel, in the given ratio.

In like manner, if three of the given lines passed through one point, as the lines CA, CB, CD, and the rectangle under QPR be to that under SPT in a given ratio, this case is with the same facility reduced Fig. 2. to the like quadrilateral thus.

Draw BE parallel to AC, that shall cut ST produced in t, and let the point F be taken, that the rectangle under CA, EF be to the square of AB in the ratio given; then CrF being drawn, Bt, or QP, will be to Tt as AC to AB, and Rr to AQ, or SP, as EF to AB; whence the rectangle under QP, Rr will be to that under Tt, SP, as that under AC, EF to the square of AB, that is, in the given ratio of the rectangle under QPR to that under SPT, and the rectangle under QPr will be to that under SPt or AQB in the quadrilateral ABCF, whose two sides AC, BF are parallel, in the same given ratio.

Now let ABCD be a quadrilateral having the two sides AC, BD parallel, with any conic section passing through the four points A, B, C, D; Fig. 3, 4, 5. also, the point E being taken in the section, and EFG being drawn parallel to AC or BD, let the ratio of the rectangle under AGB to the rectangle under FEG be given: then the conic section will be given.

Let the sides AB, CD meet in M, and draw MI bisecting AC and BD in K and L. Then the diameter of the section, to which AC and BD are lines ordinately applied, will be in the line MI; and if Fig. 3, 4. NP, QS are tangents to the section, and parallel to AC and BD, the points O, R, in which they intersect MI, will be the points of their contact, and the vertexes of that diameter. But the square of NO is to the rectangle under ANB, and the square of QR to the rectangle under AQB, as the rectangle under EGH or FEG to that under AGB, therefore in a given ratio; but the ratio of NM to NO, the same as that of QM to QR, is also given; whence the ratio of the square of NM to the rectangle under ANB, or of the square of OM to the rectangle under KOL, is given, as likewise the ratio of the square of RM to the rectangle under KRL.

Fig. 3. Now in the ellipsis the square of MO, the distance of the remoter vertex of the diameter OR from M, is greater than the rectangle under KOL; that is, the ratio given of the rectangle under FEG to that under AGB, must be greater than the ratio of the square of half the difference between AC and BD to the square of AB,

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But in the hyperbola the square of MO is less than the rectangle under KOL ; whereby the ratio of the rectangle under FEG to that under AGB , shall be less than that of the square of half the difference between AC and BD to the square of AB *. Fig. 4.

In both cases, if the point T be such, that the rectangle under MOT be equal to that under LOK , whereby MO shall be to OT in the given ratio of the square of MO to the rectangle under LOK , the given rectangle under KML will be to the rectangle under LTK (by Prop. 35, L. 7, Papp. †) in this given ratio, and therefore given; consequently the points T and O will be given. Fig. 3, 4.

In like manner, if the rectangle under MRV be equal to that under LRK , so that MR be to RV in the given ratio of the square of RM to the rectangle under LRK , the given rectangle under KML (by Prop. 22, L. 7, Papp.) will be to the rectangle under LVK in the same given proportion, whence the points V and R will be given.

Thus in both cases the points T and V will be found by applying to the given line KL a rectangle exceeding by a square, to which the given rectangle under KML shall be in the given ratio of the square of MO to the rectangle under KOL , or of the square of MR to the rectangle under KRL ; MO being to OT , and MR to RV , in that given ratio.

But in the last place, if this given ratio be that of equality, so that the square of RM be equal to the rectangle under KRL , by adding to both the rectangle under MRL , that under RML will be equal to that under KM , LR , and MR to RL as KM to LM , and the vertex R of the diameter RI will be given, the conic section being here a parabola, this diameter having thus but one vertex. Fig. 5.

Hitherto the point E , when the line EFG falls between AC and BD , is without the quadrilateral, and within the lines AB , CD , when EFG is without the quadrilateral.

But when E is within the lines AC , BD in the first case, and without in the second, the locus of the point E will be opposite sections, each passing through two angles of the quadrilateral.

When one section passes through A and C , and the other through B and D ; then if the diameter MI be drawn, as before, and to KL be applied a rectangle deficient by a square, to which the given rectangle under KML shall be in the given ratio of the square of MO to the rectangle under KOL , or of the square of MR to the rectangle under KRL , the points T and V , constituting the rectangles under KT Fig. 6.

* As the square of OM shall be greater or less than the rectangle under KOL , the square of NM will be respectively greater or less than the rectangle under ANB ; therefore the ratio of the square of NO to the rectangle under ANB , that is, of the rectangle under FEG to that under AGB , will be accordingly greater or less than the ratio of the square of NO to the square of NM , which is the same with that of the square of the difference between AK , BL to the square of AB .

† See page 511.

and under KVL, being thus found, MO will be to OT, and MR to RV, in this given ratio (by Prop. 30, L. 7, Papp.) O and T being the vertices of the diameter MI.

But the rectangles under KTL, KVL cannot be assigned, as here required, unless the ratio given for that of the square of OM to the rectangle under KOL, or that of the square of RM to the rectangle under KRL, be not less than that of the rectangle under KME to the square of half KL; that is, when the ratio of the square of ON to the rectangle under ANB, and that of the square of RQ to the rectangle under AQB, or that of the given ratio of the rectangle under FEG to that under AGB is not less than that of the rectangle under AK, BL to the square of half AB, or of the rectangle under AC, BD to the square of AB.

But if one of the opposite sections pass through A and B, and the other through C and D, the ratio of the rectangle under FEG to that under Fig. 7. AGB will be less than that of the rectangle under AC, BD to the square of AB.

For CL being drawn parallel to AB, and AD joined and continued to M, the line DM falls wholly within the section passing through C and D: therefore KM is less than KL, and the ratio of KD to KL less than that of KD to KM, that is, of BD to AB; whence BK being equal to AC, and CK to AB, the ratio of the rectangle under BKD to that under CKL, being the ratio of the rectangle under EGH, or FEG, to that under AGB, will be less than the ratio of the rectangle under AC, BD to the square of AB.

And here the point L is given; for the given rectangle under BKD is to that under CKL in the given ratio of the rectangle under HGE, or that under FEG, to the rectangle under AGB; hence CK, equal to AB, being given, KL is given, and consequently the point L.

Again, BL being joined, and NEOP drawn parallel to AB, also GEF continued to Q, as AG, equal to CQ, to FQ so will CK be to DK, and OP to EG, equal to OB, as KL to BK, consequently the rectangle under OP, AG will be to that under EG, FQ as that under KL, CK to that under KB, DK, that is, as the rectangle under AGB to that under FEG; and by combining the antecedents and consequents the rectangle under PEN will be to that under QEG in the same given ratio.

Moreover DK being to AC as KM to CM, the ratio of DK to AC, that is, the ratio of the rectangle under BKD to the square of AC, will be less than the ratio of KL to CL, or the ratio of the rectangle under CKL to that under AB, CL; therefore, by permutation and inversion, the ratio of the rectangle under CKL to the rectangle under BKD, that is, the given ratio of the rectangle under NEP to that under ANC, equal to that under GEQ, is greater than the ratio of that under AB, CL to the square of AC. And hence the opposite sections passing through the angles of the quadrilateral ABCL, whose sides AB, CL are parallel, will be given as before.

When the given ratio of the square of OM to the rectangle under LOK shall be that of the rectangle under KML to the square of half KL, whereby the given ratio of the rectangle under FEG

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to that under AGB shall be that of the rectangle under AC, BD to the square of AB, the points T and V shall unite in one, bisecting KL, and the points O and R shall also unite in one, dividing the line KLM harmonically; and then the *locus* of the point E will be each of the diagonals of the quadrilateral.

In the last place, if the diagonals AD, BC of the quadrilateral were drawn, cutting GE in I and K, and the ratio of the rectangle under KEI to that under AID were given, and not that of the rectangle under GEF to that under AGB; then the intersection of these diagonals, as L, will be in the line drawn from M bisecting AC, and BD, and the point L will fall within the quadrilateral, whereby the *locus*, when an ellipsis or single hyperbola, will be assigned by the 36th proposition of the forefaid book of Pappus: and when opposite sections, by the 30th proposition, or be reduced to the preceding cases thus.

Since KG will be to GB as CA to AB, and IG to GA as BD to AB, the rectangle under KGI will be to that under AGB, in the given ratio of the rectangle under AC, BD to the square of AB. Therefore when the ratio of the rectangle under KEI to that under AID is given, the rectangle under AID also bearing a given ratio to that under AGB, the ratio of the rectangle under KEI to that under AGB will be given, and in the last place the ratio of the rectangle under GEF to that under AGB will be given, this rectangle under GEF being the excess of that under KGI above that under KEI*. And thereby the sections will be determined, as before.

AND thus may the *locus* of the point sought be assigned in all the cases of this ancient problem, which Sir Isaac Newton has distinctly explained. The other cases, he has alluded to, may be treated, as follows.

When three of the given lines shall be parallel, as AC, BD, and HI, the fourth line being AB, and KELM being parallel to AB, Fig. 9. the ratio of the rectangle under KEL to the rectangle under EG and EM shall be given; that is, three points, A, B, and H being given in the line AB, with the line GE insinuating on AB in a given angle, that the rectangle under AGB shall be to that under GH and GE in a given ratio: then take AN equal to BH, and draw NO parallel to AC, BD, and HI.

Then if NP be drawn, that PO be to ON in the given ratio, NP will be given in position, and PO will be to ON, that is, EG, as the rectangle under KEL to that under EM, EG; so that the rectangle under KEL will be equal to that under PO, EM. But the rectangle under OKM is equal to the excess of that under OEM above that under KEL†; therefore the rectangle under OKM, or that under NAH, or under NBH, is equal to that under EM and the excess of OE above OP, that is, to the rectangle under PEM; the point E therefore is an hyperbola described to the given asymptotes PN, MH, and passing through A and B.

* By Prop. 193, Lib. 7, Papp.

† By Prop. 194, Lib. 7, Papp.

Again, if two of the given lines only are parallel, but the rectangles otherwise related to them, than as above. Suppose the ratio of the rectangle under AG, EF to that under BG, GE is given. Let CD meet AB in L, and let HEI, MFN be drawn parallel to AB, and LK parallel to AC and BD. Then the parallelogram EM will be to the parallelogram EB in the given ratio. Take AO to OB in that ratio, and draw OP parallel to AC and BD. Here the point O will be given, and the parallelogram PA will be in the given ratio to the parallelogram PB; whence AB will be to BO as the parallelogram BH to the parallelogram BP, and as the difference between the parallelograms EM and EB to the parallelogram EB, consequently as the parallelogram GM to the parallelogram PG; therefore the ratio of the rectangle under AG, FG to the rectangle under EG, EP or OG will be given; and in the last place the ratio of FG to GL being given, the ratio of the rectangle under AG and GL to that under EG, OG will be given. And thus three points, A, L, O, will be given with GE insisting on AB in a given angle, as in the preceding case.

Moreover, AC and BD being parallel, AB and CD may be also parallel. And then, when the ratio of the rectangle under AGB to that under GEF is given, the determination of the *locus* is so obvious as not to have required a distinct explanation. But when the rectangle under AG, EF bears a given ratio to that under BG, GE; let the diagonals AD, BC be drawn, and HELK drawn parallel to AD. Then the rectangle under HEL will be to that under KEI in the same given ratio; and if CM be taken to MB in the same ratio, the lines MNP, MOQ drawn, the first parallel to AC, BD, and the other parallel to AB, CD will be given in position, and the diagonal BM will bisect both IK, NO, and HL; therefore the rectangle under HEL being to that under KEI as MC to MB, that is, as NH to NK, here by division the rectangle under HEL will be to that under IHK* as NH to HK; therefore equal to that under NH and IH or KI. But the rectangle under NEO is equal to the sum of the rectangles under HNL and under HEL†; therefore the rectangle under NEO is equal to that under NH, NK, equal to that under APD, that is, equal to that under PAQ, or that under PDQ, the diagonal BM bisecting both PQ and AD. But thus the point E is an hyperbola described to the asymptotes MN, MO, and passing through A and D.

THE determination of this *locus* for three lines is solved almost explicitly by Appollonius in the three last propositions of his third book of Conics. For if the three lines proposed were AB, AC, BC, and the

point sought D, that the ratio of the rectangle under ED (the line EF being drawn parallel to BC) should be in a given ratio to the square of a line drawn from D to BC in a given angle, the square of which line will be in a given ratio to the

* By the prop. of Papp. before cited. † By the same.

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rectangle under BE, CF; then if BH, CI are drawn parallel to AC and AB respectively, also BDL, CDK drawn through D, the square of BC will be to the rectangle under BK, CL as the rectangle under DF, DE to that under CF, BE.

Hence if the ratio of the rectangle under DF, DE to the square of a line drawn from D on BC in a given angle, is given; the square of this line being in a given ratio to the rectangle under CF, BE, the ratio of the rectangle under BK, CL to the square of BC will be given; whence a conic section passing through D will in all cases be given.

In the first place let the point D be within the angle BAC. Then if BC be bisected by the line AM, this will be a diameter to Fig. 12. the conic section, which shall touch BA, AC in the points B, C, and BC will be ordinately applied to that diameter; the vertex of this diameter being N, the given ratio of the rectangle under BK, CL to the square of BC will be compounded of the ratio of the square of MN to the square of NA, and of the ratio of the rectangle under BAC to the fourth part of the square of BC; and thus the line AM will be divided in N in a given ratio, and the point N, one vertex of the diameter, to which BC is ordinately applied, will be given.

If AN be equal to NM, the point N will be the only vertex of this diameter, and the section will be a parabola.

Otherwise by taking the point O in AM extended, so that the ratio of AO to OM be the same with that of AN to NM, the point O will be the other vertex of the diameter.

And here if the ratio of AN to NM be that of a greater to a less, the point O will fall beyond M from A within the angle BAC, the conic section being an ellipsis.

But if the ratio of AN to NM be that of a less to a greater, the point O will fall on the other side of A, and the section will be an hyperbola.

And in this case if the opposite section be drawn, that also will be the *locus* of the point D within the angle vertical to Fig. 13. the angle BAC.

In the last place, if D be in either of the collateral angles, AM drawn as before will contain a secondary diameter in opposite sections, one of which shall touch BA in B, and the other CA in C. Then Fig. 14. if one of these sections pass through D, the sections will be given.

For here PAQ being drawn through A parallel to BC, the given ratio of the rectangle under CL, BK to the square of BC will be the same with that of the given rectangle under BAC to the square of AP: therefore AP is given, and thence the sections. For let RS be the secondary diameter, to which BC is ordinately applied, and T the center of the opposite sections. Then the square of BM will be to the rectangle under AMT as the square of the transverse diameter conjugate to the secondary diameter RS to the square of this secondary diameter; and if a line were drawn from M to P, this would touch the hyperbola BP in P, and the square of AP will be to the rectangle under MAT in the

same ratio; therefore the given ratio of the square of MB to the square of AP will be that of the rectangle under AMT to the rectangle under MAT, or the ratio of MT to AT; consequently the ratio of MT to AT is given, and thence the point T. But also the diameter RS is given in magnitude, the square of RT or of ST being equal to the rectangle under MTA; whence in the last place the transverse diameter conjugate to this is also given; for the square of this diameter is to the square of RT as the given square of BM to the rectangle under AMT now also given.

Fig. 15. But a more simple case may also be proposed in three lines, when the ratio of the rectangle under EDF should be equal to the rectangle under a given line, and that drawn from D to BC in a given angle.

This line will bear, both to BE and FC, a given ratio, and the rectangle under EDF will be in a given ratio to the rectangle under the given line and EB or CF.

Let the line given be H, and take MB and NC, that the rectangle under MBC, and that under BCN be to that under BA and H in the given ratio of the rectangle under EDF to that under BE and H, BM and CN being equal. Then draw from M and N lines parallel to BA, CA, which shall intersect EF in K and L, whereby, MK cutting CA in I, the rectangle under MBC will be to that under BA and H as the rectangle under BMC to that under MI and H, and also as the rectangle under EKF to that under KI and H, that is, as the rectangle under EDF to that under H and BE or MK, whence by adding the antecedents and consequents the rectangle under KDL will be to the rectangle under H and MI in the same given ratio, which is also that of the rectangle under BMC to the same rectangle under H and MI: the point D therefore is in an hyperbola passing through B and C, having for asymptotes the lines MK and NL given in position, the rectangle under KDL being equal to that under BMC, or that under MBN.

If the two lines AB and AC are parallel, the *locus* may be known to be a parabola by the last proposition of the fourth book of Pappus.

But if BC were parallel to one of the other, the *locus* will be an hyperbola, as the preceding, but assigned by a shorter process.

Fig. 16. Suppose the given lines to be AE, AF, and BC parallel to AF. And let the rectangle under EDF be equal to that under DG, and the given line H, the line EG making given angles with AE, AF. Here take EI equal to H, and deduct from both the rectangles that under EI or H, and DF, whereby will be left the rectangle under IDF equal to that under H and FG, both whose sides are given. Draw therefore IK parallel to AE, and the rectangle under IDF will be equal to this given rectangle, the given lines KI, AF being the asymptotes to the hyperbola passing through D.

Coroll. If LM be drawn through B parallel to EF, LB shall be equal to FG, and BM equal to EI or H, whereby the hyperbola opposite to that passing through D will pass through B.

SCHOLIUM.

The propositions of Pappus, which have been referred to in page 500, 501, 504, l. 2, are given by him, among others, for Lemmas subservient to the lost treatise of Appollonius *De sectione determinata*, and the four here cited respect and comprehend all the cases of the problem, where three points are given in any line, and a fourth is required such, that the rectangle under the segments of the proposed line intercepted between the point sought, and two of the given points, shall bear a given ratio to the square of the segment terminated by the third point.

The cases indeed of the problem, from the diversity of situation in the points given to the point sought, and to one another, are in number six. The given extreme of the segment to constitute the square may either be without the other two given points, or between them. And when it is without, the point sought may be required to be taken without them all, either on the side opposite to the given extrema of the segment to constitute the square, which will be one case, or it may be required to fall on the same side, which will be a second case. If it be required to fall between this point and the other two, this will be a third case. A fourth case will be, when the point sought shall be required to fall between the other two points. Also when the given extreme of the segment to constitute the square lies between the other two given points, the point sought may be required to fall, either there also, or without, composing the 5th, and 6th cases.

The propositions in Pappus, referring to these cases, though but four in number, suffice for them all, each proposition being applicable to the problem two ways. For instance, the thirty-fifth proposition, as expressed by Pappus, is this, being the first above cited. Three points C, D, E being taken in the line AB, so that the rectangle under ABE be equal to that under CBD, AB is to BE as the rectangle under DAC to that under CED. Now AB is to BE, both as the square of AB to the rectangle under ABE, and as the rectangle under ABE to the square of BE. Therefore if the ratio of AB to BE be given, the ratio of the square of AB to the rectangle under CBD will be given, which is the first of the cases above described, and also the ratio of the rectangle under CBD to the square of BE given, which is the second case. In both cases the rectangle under DAC will be to that under CED in the given ratio of AB to BE. But in the first the rectangle under DAC will be given, and the point E in the rectangle under CED to be found by applying a rectangle, which shall bear a given ratio to the given rectangle under DAC to the given line CD exceeding by a square; and in the second case the rectangle under CED is given, and A in the rectangle under DAC to be found by applying to the given line CD a rectangle exceeding by a square, which shall bear a given ratio to the rectangle under CED now given;

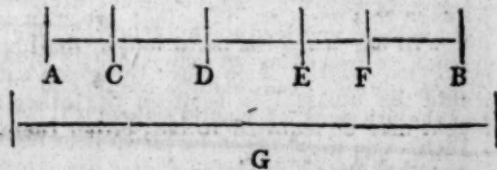
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whence by the ratio of AB to BE given the point B will be found in both cases.

The 22d proposition either way applied refers to the 3d case only, the 30th relates both to the 4th and 5th, and the 36th proposition to the remaining 6th.

The 45th, and other following propositions, are accommodated to the solution of Appollonius's problem, when four points are given, and a fifth required, which with the given points shall form four segments such, that the rectangle under two shall bear a given proportion to the rectangle under the other two. The various cases of this problem appear to have been the subject of the second book of the mentioned treatise of Appollonius; and, according to the character given by Pappus of those propositions, these lemmas serve to reduce them to problems in the first book, not those above mentioned, but those, where three points being given, the rectangle under the segments included by two, and a fourth point shall bear a given proportion to the rectangle under the segment formed by the third point and a given line.

For instance, the 46th proposition is this; in the line AB four points A, C, E, B being given; and the point F assumed between E and B; also D taken, according to the 41st proposition, that the rectangle under ADC be equal to that under BDE; if G be equal to the sum of AE, CB, the rectangle under AFC, together with that under EFB, will be equal to the rectangle under G and DF.



Here if it were proposed to find the point F, that the ratio of the rectangle under AFC to that under EFB should be given, the ratio of the rectangle under AFC to that under DF and the given line G would be given.

But this analysis may be carried on to a compleat solution of the problem thus. If CN be taken to G in the given ratio of the rectangle under AFC to that under DF and G, the point N will be given, and the rectangle under AF, CN will be to that under AF, G in this ratio of CN to G; consequently the



excess of the rectangle under AF, CN above that under AFC, that is, the rectangle under AFN, will be to the excess of the rectangle under AF and G above that under DF and G, or the given rectangle under AD, G, in the same given ratio, and in the last place the rectangle under AFN will equal the given rectangle under AD and CN.

Here I have chosen this proposition in particular, because the case of the problem, to which it is subservient, is subject to a determination, when FN shall be equal to AF. And then the rectangle under AFN being equal to that under AD and CN, as CN to FN so is AF to AD, and by division as CF to FN so DF to AD; therefore when AF is equal to FN, CF will be to AF as FD to AD: consequently CD to FD as FD to AD, and the square of DF equal to the rectangle under ADC, when the problem admits of a single solution only, wherein the rectangle under AFC will bear to that under EFB a less ratio than in any other situation of the point F between E and B.

Moreover CN is to G as the rectangle under AFC to the sum of the rectangles under AFC and EFB; therefore FN being equal to AF, when the problem is limited to this single solution, the rectangle under AFC shall be to the rectangles under AFC and EFB together as the sum of AF and FC to G, which is equal to the sum of AE and CB; whence by division the ratio of the rectangle under AFC to that under EFB, when the problem is limited to this single solution, will be that of the sum of AF and CF to the excess of FB above EF.

Thus directly do these lemmas correspond with Apollonius's first mode of solution, and lead to the general principle of applying to a given line a rectangle exceeding or deficient by a square, which shall be equal to a space given. This being a simple case of the 28th and 29th propositions of the 6th book of Euclid's elements, admits of a compendious solution. Such a one is exhibited by Snellius in his treatise on these problems, (in Apollon. Batav.) and Des Cartes has exhibited another more contracted in its terms, but not therefore more useful. It may also be performed thus.

Fig. 17. If upon a given line AB any triangle ACB be erected at pleasure; then if the legs CA, CB, whether equal or unequal, be continued to D and E, that the rectangles under CAD and CBE be each equal to the given space, and a circle be described through C, D, E, cutting

Fig. 18. AB extended in F and G, the rectangle under BFA and BGA will each be equal to the space given. Also if in the legs CA, CB the rectangles under CAD and CBE be each taken equal to the space given, and a circle in like manner be described through C, D, E, cutting AB in F and G, the rectangles under AFB and AGB will each be equal to the given space. Here it is evident, that the space given must not exceed the square of half AB, when equal, the circle will touch AB in its middle point.

POSTSCRIPT.

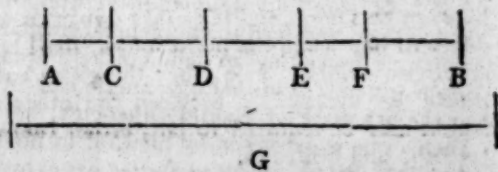
AS this application to a given line of a rectangle exceeding or deficient by a square, or the more general problem treated of in the sixth book of the elements, of applying a space to a line so as to exceed or be deficient by a parallelogram given in species, is the most obvious result, to which the analysis of plane problems, not too simple to require this construction,

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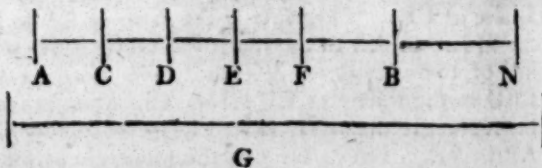
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POSTSCRIPT.

AS this application to a given line of a rectangle exceeding or deficient by a square, or the more general problem treated of in the sixth book of the elements, of applying a space to a line so as to exceed or be deficient by a parallelogram given in species, is the most obvious result, to which the analysis of plane problems, not too simple to require this construction,

leads; so the descriptions of the conic sections here treated of, stand in the like stead in regard to the higher order of problems styled solid from the use of the conic sections deemed necessary for their genuine solution. And these are the only modes of solution, the modern algebra, which grounds its operations on one or two elementary propositions only, naturally leads to. But as the form of analysis amongst the antients, by expatiating through a larger field, often was found to arrive at conclusions much more concise and elegant, than could offer themselves in a more confined track; the ancient sages in geometry, that the solid order of problems might not want this advantage, fought out that copious and judicious collection of properties attending the conic sections, which, with some useful additions from later writers, have been handed down to us.

And as the advantages of this ancient system of analysis cannot be too much inculcated in an age, wherein it has been so little known, and almost totally neglected, permit me, Sir, to close this address to you with an example in each species of problems.

Were it proposed to draw a triangle given in species, that two of its angles might touch each a right line given in position, and the third angle a given point. It is obvious, how difficult it would be to adopt a commodious algebraic calculation to this problem; notwithstanding it admits of more than one very concise solution, as follows.

Fig. 19. Let the lines given in position be AB, AC and the given point D, the triangle given in species being EDF.

In the first place suppose a circle to pass through the three points A, E, D, which shall intersect AC in G. Then EG, DG being joined, the angle DEG will be equal to the given angle DAC, both inscribing on the same arch DG; also the angle EDG is the complement to two right of the given angle BAC: these angles therefore are given, and the whole figure EFGD given in species. Consequently the angle EGF, and its equal ADE will be given together with the side DE of the triangle in position.

Fig. 20. Again, suppose a circle to pass through the three points A, E, F, cutting AD in H, and EH, FH joined. Here the angle EFH will be equal to the given angle EAH, and the angle FEH equal to the given angle FAH. Therefore the whole figure EHFD is given in species, and consequently the angle ADE, as before.

In the last place, suppose a circle to circumscribe the triangle, and intersect one of the lines, as AC, in I. Here DI being drawn, the angle DIF will be equal to the given angle DEF in the triangle; consequently DI is inclined to AC in a given angle, and is given in position, as also the point I given; whence IE being drawn, the angle FIE will be the complement of the angle EDF in the triangle to two right. Therefore IE is given in position, and by its intersection with the line AB gives the point E, with the position of DE, and thence the whole triangle, as before.

Here it may be observed, that the angle D of the triangle EDF given in species touching a given point D, and another of its angles touching AC, the line IE here found is the locus of the third angle E.

Again, in the astronomical lectures of Dr. Keil, it is proposed to find the place of the earth in the ecliptic, whence a planet in any given point of its orbit shall appear stationary in longitude, and a solution is given from the late eminent astronomer, Dr. Halley, upon the assumption, that the orbit of the earth be considered as a circle concentric to the sun.

But for a compleat solution of this problem let the following lemma be premised.

The velocity of a planet in longitude bears to the velocity of the earth the ratio, which is compounded of the subduplicate ratio of the *latus rectum* of the greater axis of the planet's orbit to the *latus rectum* of the greater axis of the earth's orbit, of the ratio of the cosine of the angle, which the orbit of the planet makes with the plane of the ecliptic, to the radius, and of the ratio of a line drawn in any angle from the center of the sun to the tangent of the orbit of the earth at the point, wherein the earth is, to a line drawn in the same angle from the sun to the tangent of the orbit of the planet projected upon the plane of the ecliptic at the place of the planet in the ecliptic.

Let A be the sun, BC the orbit of any planet, DE the same projected on the plane of the ecliptic, FG being the line of the nodes, B the place of the planet in its orbit, D its projected place: then the plane through B and D, which shall be perpendicular to both the planes BC and DE, intersecting those planes in BH, DH, the lines BH DH will be both perpendicular to the line of the nodes, and the angle BHD the inclination of the orbit to the plane of the ecliptic. But tangents drawn to BC and DE at the points B and D respectively will meet the line of the nodes, and each other in the same point I, and the velocity of the planet in longitude will be to its velocity in the orbit BC, as DI to BI.

Now from the point A let AK fall perpendicular on BI, and AL be perpendicular to DI: then the ratio of DI to IB will be compounded of the ratio of DI to DH, or of AI to AL, of the ratio of DH to BH, and of that of BH to BI, that is, of AK to AI. But DH is to BH as the cosine of the inclination of the orbit to the radius, and the two ratios, that of AI to AL, and that of AK to AI, compound the ratio of AK to AL: therefore the velocity of the planet in longitude is to the velocity in its orbit in the ratio compounded of that of the cosine of the inclination of the planet's orbit to the radius, and that of AK to AL.

Moreover the ratio of the velocity of the planet in B to the velocity of the earth in any point of its orbit is compounded of the subduplicate of the ratio of the *latus rectum* of the greater axis of the planet's orbit to the *latus rectum* of the greater axis of the earth's orbit, and of the ratio of the perpendicular let fall from the sun on the tangent of the earth's orbit at the earth to AK, the perpendicular let fall on the tangent of the planet's orbit at B. Therefore the velocity of the planet in longitude, when in B, to the velocity of the earth in any point of it's orbit is compounded of the subduplicate ratio of the *latus rectum* of the greater axis of the planet's orbit, to the *latus rectum* of the greater axis of the earth's orbit, of the ratio of the co-sine of the inclination of the planet's orbit to the radius, and of the ratio of the foresaid perpendicular on the

tangent of the earth's orbit to AL, the perpendicular on DI: these perpendiculars being in the same ratio with any lines drawn in equal angles to the respective tangents.

This being premised, the place of a planet in the ecliptic being given, the place of the earth whence the planet would appear stationary in longitude, may be assigned thus.

A denoting the sun, let B be a given place of any planet in its orbit projected orthographically on the plane of the ecliptic, CB the tangent to

Fig. 23. the planet's projected orbit at the point B, which will therefore be given in position. Also let DE be the orbit of the earth, and the point D the place of the earth, whence the planet would appear stationary in longitude at B.

Join AB, and draw a tangent to the earth's orbit at the point D, which may meet CB in F, and the line AB in G; draw also AH making with DF the angle AHD equal to that under ABC. Then the point D being the place, whence the planet appears stationary in longitude, as FB to FD so will the velocity of the planet in longitude in B be to the velocity of the earth in D, this velocity of the planet in B being also to the velocity of the earth in D in the ratio compounded of the subduplicate of the ratio of the *latus rectum* of the greater axis of the planet's orbit, to the *latus rectum* of the greater axis of the orbit of the earth, of the ratio of the co-sine of the inclination of the planet's orbit to the plane of the ecliptic to the radius, and of the ratio of AH to AB: therefore the ratio of FB to FD will be compounded of the same ratios: and if I be taken, that the ratio of AB to I be compounded of the two first of these, I will be given in magnitude, and the ratio of FB to FD will be compounded of the ratio of AB to I, and of AH to AB. Whence FB will be to FD as AH to I; and the angles CBA, or FBG, and AHG being equal, whereby FG will be to FB as AG to AH, by equality FG will be to FD as AG to I, and DK being drawn parallel to FB, BG will be to BK as FG to FD, and therefore as AG to I.

But now, as this problem may be distributed into various cases, in the first place consider the earth as moving in a circle concentric to the sun, and likewise CB, the tangent to the planet's orbit, perpendicular to AB. But here DK also will be perpendicular to AB, and AB meeting the earth's orbit in L and M, the rectangle under KAG will be equal to the

Fig. 24. square of AM. But BG being to BK as AG to I, if BN be taken equal to I, BG will be to BK as AG to BN, and AB to KN also as AG to BN, and the rectangle under NK, AG equal to that under AB and I: therefore the rectangle under KAG being equal to the square of AM, NK will be to KA as the rectangle under AB, I to the square of AM, that is, in a given ratio, and KD with the point D will be given in position.

Again, when CB is not perpendicular to LM, let DO be perpendicular to LM. Then the rectangle under OAG will be equal to the square

Fig. 25. of AM. But BN being taken equal to I, as before, the rectangle under NK, AG will be equal to that under AB, I; whence NK will be to AO in the given ratio of the rectangle under AB, I to the square of AM. Therefore NP being taken to PA

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in that ratio, the point P will be given, and KP, the excess of NP above NK, will be to PO, the excess of AP above AO, in the same ratio. Hence, as DK is parallel to CB and DO perpendicular to LM, the triangle KOD is given in species, and if PD be drawn, the angle OPD will be given; for the co-tangent of the angle OKD will be to the co-tangent of the angle OPD, as KO to OP, that is, as the rectangle under AB, I together with the square of AM to the square of AM, and hence the point D is given by the line PD drawn from a given point P in a given angle APD; and if AD be drawn, AD will be to AP as the sine of the angle APD to the sine of the angle PDA; this angle therefore is given, and the angles APD, PDA being given, the angle PAD is given.

Corol. Here, where the orbit of the earth is supposed a circle, the ratio of I to AB, that is, of the rectangle under AB, I to the square of AB, will be compounded of the subduplicate ratio of AM, the semidiameter of the earth's orbit, to half the *latus rectum* to the greater axis of the planet's orbit, and of the ratio of radius to the co-sine of the inclination of the planet's orbit to the plane of the ecliptic; and adding on both sides the ratio of the square of AB to the square of AM, the ratio of the rectangle under AB, I to the square of AM will be compounded of the ratio of the square of AB to the rectangle under AM and the mean proportional between AM and the half of this *latus rectum* of the planet's orbit, and of the ratio of the radius to the co-sine of the inclination of the planet's orbit.

In the next place, though the earth's orbit is not a circle concentric to the sun; yet if the projection of the planet falls on the line perpendicular to the axis of the earth's orbit, the point A will still bisect LM.

In this case draw the points L and M tangents to the ellipsis meeting in P, from whence through D draw PD meeting the ellipsis again in Q, and intersecting LM in O. Here if a tangent be drawn to the ellipsis in Q, it will meet the tangent at D on the line LM in the point G. Fig. 26.

Now LG will be to GM as LO to OM, and the point A bisecting LM, the rectangle under GAO will be equal to the square of AM. But BG is to BK as AG to I. Therefore BN being taken equal to I, AB will be to KN as AG to I, and the rectangle under AB, I equal to that under AG, KN; whence AO being to KN as the rectangle under GAO to that under AG and KN, AO will be to KN as the given square of AM to the rectangle under AB and I, also given.

Draw RP parallel to CB, and take PS to AP, also NT to AR in this given ratio inverted. Then will the points T and S be both given, also AO will be to KN, and RO to KT, as AR to NT, that is, as AP to PS. Therefore if TV be drawn parallel to CB, that is, to KD, and VS parallel to LM, these lines will be both given in position; and WDX being also drawn parallel to LM, WD will be equal to KT, and RO being to KT, as AP to PS, DY will be to WD as XP to PS, and by composition YW to WD as XS to PS, and the given rectangle under YW, or SV, and PS equal to that under WD, and XS. Whence SV being parallel to LM,

the point D will be in an hyperbola passing thro' P, and having for asymptotes the lines VS, VT given in position.

But if the projection of the planet fall on the axis of the earth's orbit, or the same continued, AB extended to the earth's orbit in L and M will be the axis of that orbit.

If also CB should be perpendicular to AB, KD would be ordinately applied to LM; and the point R being taken, that Q being the center of the orbit, the rectangle under AQR be equal to the square of QM, the same will be equal also to the rectangle under GQK; whence as GQ to AQ so RQ to QK, and AG to AQ as KR to QK. But as above, BG being to BK as AG to I, and BN taken equal to I, BG will be to BK as AG to BN, and AB to KN also as AG to BN or I. Therefore if NS be taken to AB as I to AQ, by equality NS will be to NK as AG to AQ, that is, as KR to QK; and in the last place NS to KS as KR to QR, that is, the rectangle under SKR equal to the given rectangle under NS, QR; whence the point K, the position of KD, and thence the point D will be given.

But if DK be not ordinately applied to LM, let DO be ordinately applied to LM. Then here the rectangle under AQR, equal to the square of QM, will be equal to that under OQG, and GQ to AQ as OR to OQ. But BN being now also taken equal to I, and NS to AB as I to AQ, AB will be here in like manner to KN as AG to I, and NS to KN as AG to AQ: therefore NS will be to KN as OR to OQ, and by conversion NS to KS as OR to QR. But NS and QR being both given in magnitude, if SP be taken to NS as QR to PR, the point P will be given, and also by equality SP will be to KS as OR to PR; whence if RV be drawn parallel to DO, and ST to KD, both RV and ST will be given in position, one passing through the given point R, parallel to the ordinates applied to the axis LM, and the other through the point S also given, and parallel to KD or CB: also DTV being drawn parallel to ML, DT will be equal to KS and DV equal to OR, therefore as SP to DT so DV to PR, and the rectangle under SPR equal to that under TDV, consequently the point D in an hyperbola passing thro' P, and having for asymptotes the lines ST, RV given in position.

In the last place when the line LM drawn through the sun in A, and the projected place of the planet in B, is neither the axis of the earth's orbit, nor bisected in A, the tangents to the points L, M being drawn to

Fig. 29. meet in P, let LM be bisected in Q, and the point R taken that the rectangle under AQR be equal to the square of QM, whereby PDO being drawn, the rectangle under AQR shall be equal to that under OQG, and QG to AQ as QR to QO, or by composition AG to AQ as OR to QO. Therefore if NB be here also taken equal to I, and NS to AB as I to AQ, AB being as before, to NK as AG to I; by equality NS will be to NK as AG to AQ, that is, as OR to QO. Whence by conversion NS will be to KS as OR to QR; and if PT be drawn parallel to CB and SV be here taken to NS as QR to TR, by

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equality SV will be to KS as OR to TR and also by conversion SV to KV as OR to OT . Moreover SV will be given in magnitude, and the point V given; therefore VW drawn parallel to CB , or KD , will here be given in position. But $WDX Y$ being also drawn parallel to RV , SV will be to KV , or DW , as YD to XD , and YZ being taken equal to the given line SV , YZ will be to DW as ZD to XW , equal to TV , and the given rectangle under YZ , TV equal that under WDZ . Therefore ΓZ being drawn parallel to RP , RT , and its equal YZ , being given, the line ΓZ is given in position, and the point D in an hyperbola having for asymptotes VW , ΓZ , and passing through P .

Thus is this problem in all cases solved either by a right line, or an hyperbola given in position, which shall intersect the projected orbit in the point sought. For though in each case the projection of the planet has here been considered as within the orbit of the earth, the form of argumentation will be altogether similar, were the projection of the planet without. And this is agreeable to the method, I have pursued throughout this discourse, where I have always accommodated the expression to one situation only of the terms given and sought in each article; the variation necessary for the other cases, when one has been duly explained, being sufficiently obvious.

In the 5th volume of the Commentaries of the Royal Academy at Petersbourg is given an algebraical computation for a general solution of this problem in the orbits of any two planets projected on the plane of the ecliptic; but with this oversight of applying to the projected orbits a proposition from Dr. Keil's Astronomical Lectures, which relates to the real orbits*.

However from the geometrical solution now given a calculation for assigning the point D may be formed without difficulty. LDM being the orbit of the earth, A is the focus, and RP perpendicular to the axis. Let the axis be ab meeting RP in c , ΓZ in d , PT in e and WV in f . Then the angle aAM is given, being the distance between the heliocentric place of the planet in the ecliptic from the earth's aphelion. Also PT being parallel to CB , the angle ATe , and consequently the angle AeT , will in like manner be given, whence the points Γ , R , T , V being given, as in the solution above, the points d , c , e , and f will be given, the triangles ARc , ATe , being given in species, and similar respectively to the triangles $A\Gamma d$, and AVf . Also the rectangle under WDZ being equal to that under $R\Gamma$, VT , if DK be continued to the axis in g , and Dh be drawn parallel to PR , the rectangle under fg , hd is equal to that under fe , dc , and both being deducted from the rectangle

* The demonstration of Dr. Keil's proposition proceeds on the known property in the planets of having their periodic times in the sesquialterate ratio of the axes of their orbits, which confines the proposition to the real orbits; for in each planet the periodic time through the projected orbit is the same, as through the real, though the axis in one be not equal to the axis of the other.

under $f h d$ the excess of the rectangle under $f h d$ above that under $f e$, $d c$ will be equal to that under $g h d$, so that this difference will be a mean proportional between the square of $h d$ and the square of $h g$, which is in a given ratio to the square of $h D$, and therefore in a given ratio to the rectangle under $a h b$, $D h$ being ordinately applied to the axis $a b$.

Thus a biquadratic equation may be formed, whereby the point h shall be found, and thence the point D , whose distance from A is to $h c$ as the eccentricity of the earth's orbit to half its axis.

Therefore I shall only observe farther, that here occurs an obvious question, what, in so extended a search for principles leading to the solution of any problem, as the ancient analysis admits of, can conduct to the most genuine upon each several occasion. But for this end, where commodious principles do not readily offer themselves, the most general means is to consider first simple cases of the problem in question, and from thence to proceed gradually to the more complex, as has been here done in the present problem, where the several preceding cases lead one after another to the points and lines required for the last case, wherein the problem is stated in its most extensive form:

Concise Rules for computing the Effects of Refraction and Parallax in varying the Apparent Distance of the Moon from the Sun or a Star; also an easy Rule of Approximation for computing the Distance of the Moon from a Star, the Longitudes and Latitudes of both being given, with Demonstrations of the same: By the Rev. Nevil Maskelyne, A. M. Fellow of Trinity College, in the University of Cambridge, and F. R. S.

Read Nov. 15, 1764. **T**HE following rules, excepting one, are the same which I have already communicated to the Royal Society, but without demonstration, in a letter to the reverend Dr. Birch from St. Helena, containing the results of my observations of the distance of the Moon from the Sun and fixed stars, taken in my voyage thither, for finding the longitude of the ship from time to time; since printed in Part II. Vol. LII. of the Philosophical Transactions for 1762. The two rules for the correction of refraction and parallax I have also since communicated to the public in my British Mariner's Guide to the discovery of longitude from like observations of the Moon; and have added in the Preface a rule for computing a second but smaller correction of parallax, necessary on account of a small imperfection lying in the first rule derived from the fluxions of a spherical triangle. To the rules I have here subjoined their demonstrations.

With respect to the usefulness of these rules, I cannot but entertain hopes that they will appear more simple and easy than any yet proposed for

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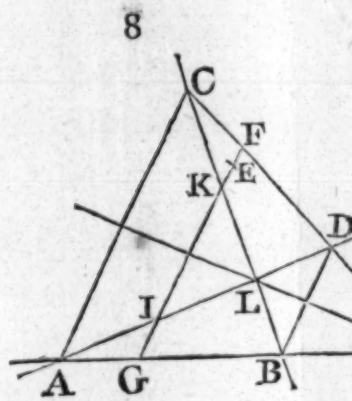
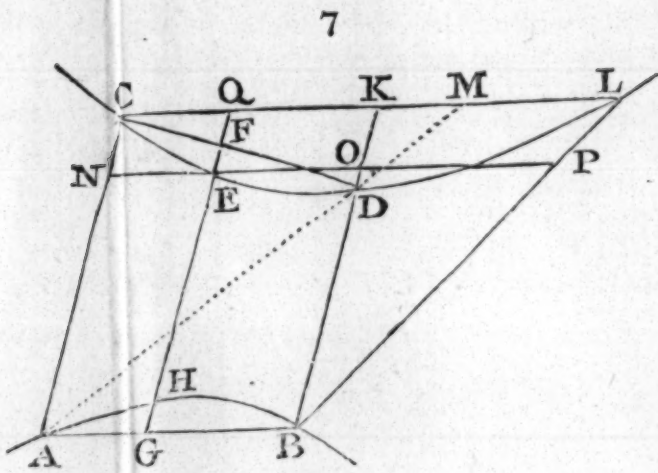
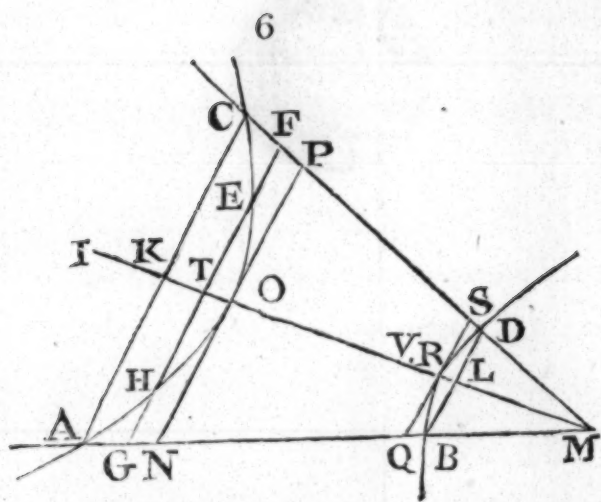
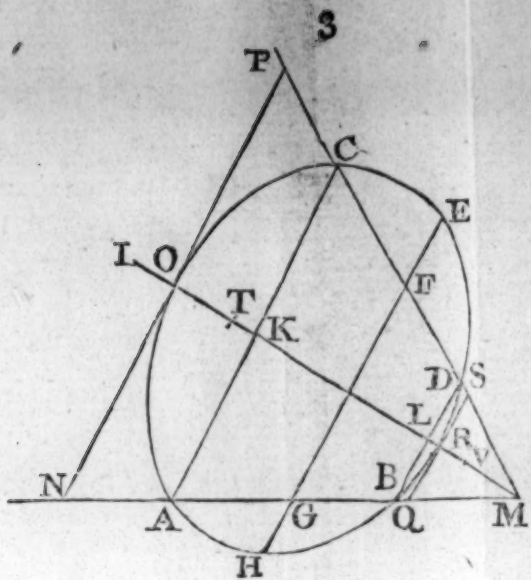
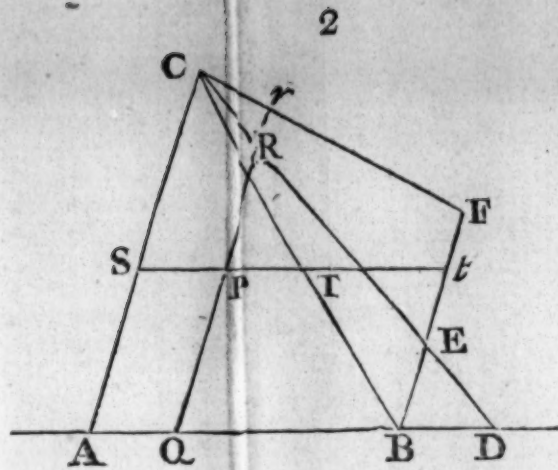
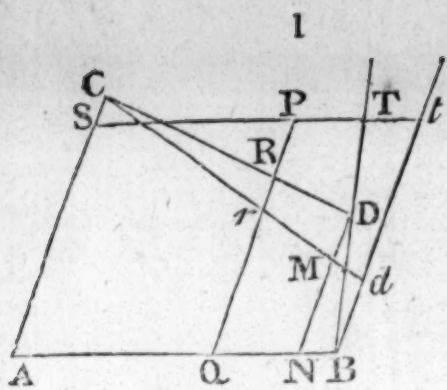
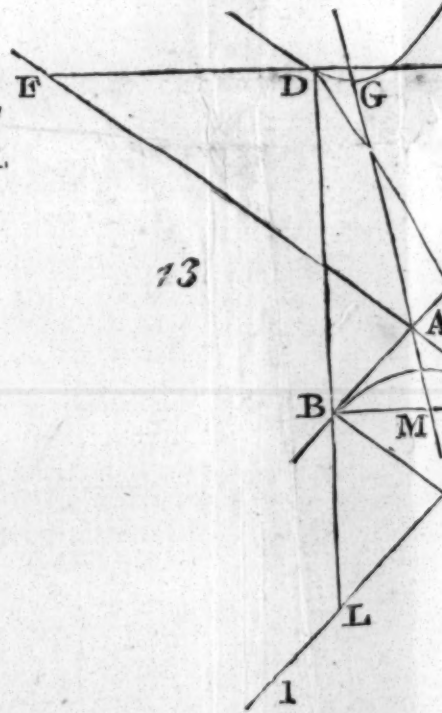
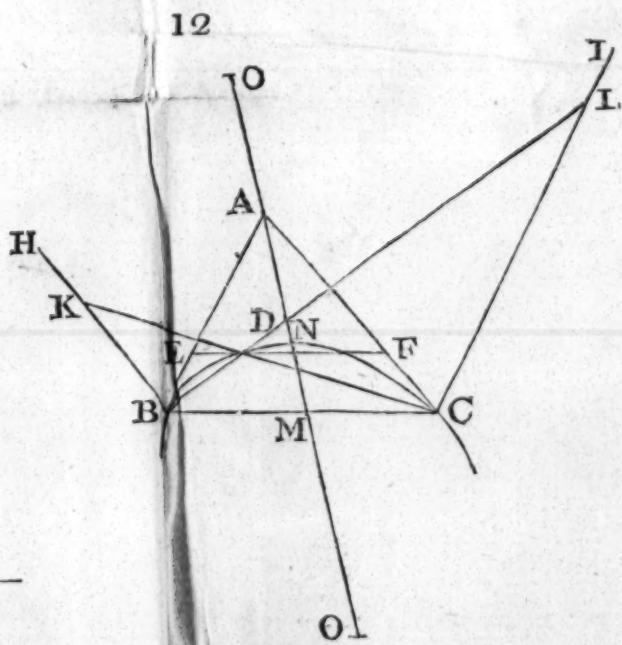
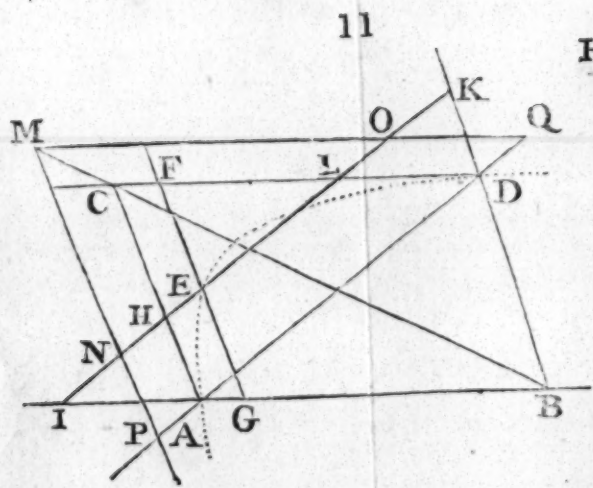


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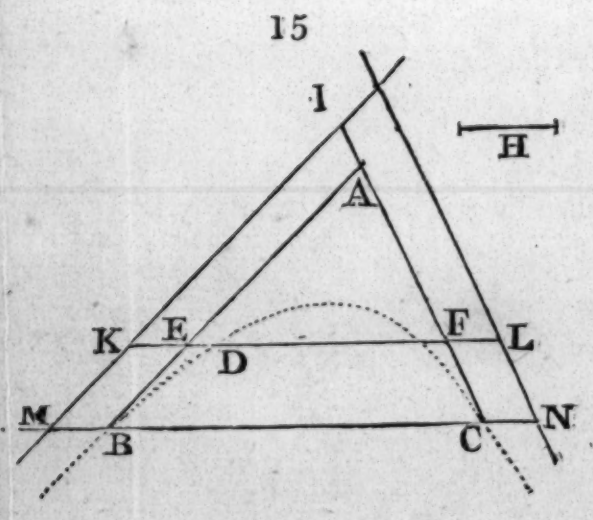
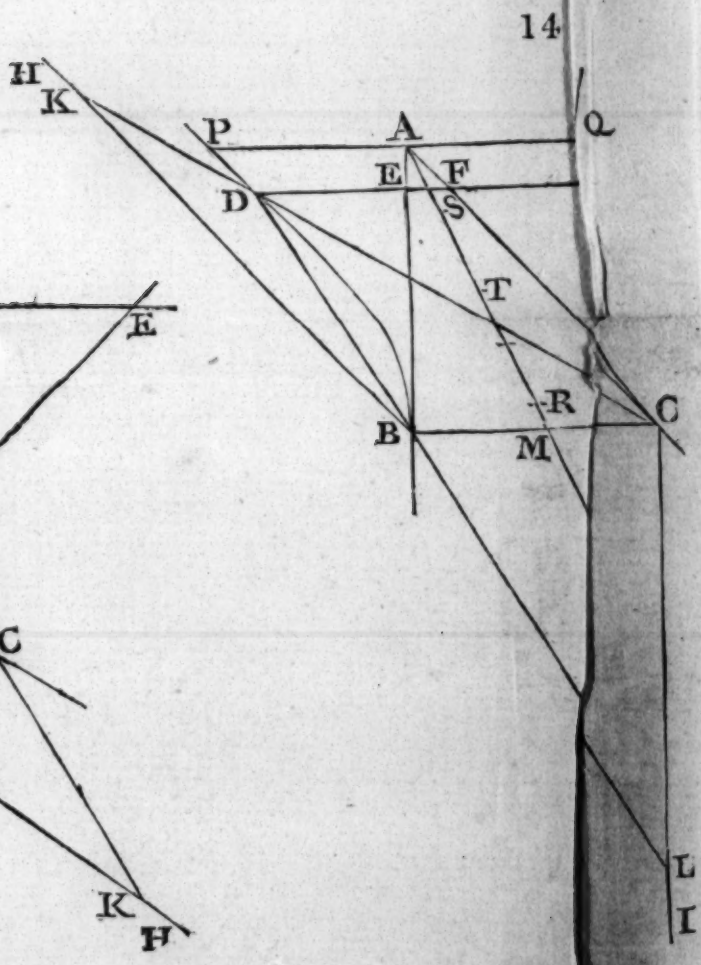
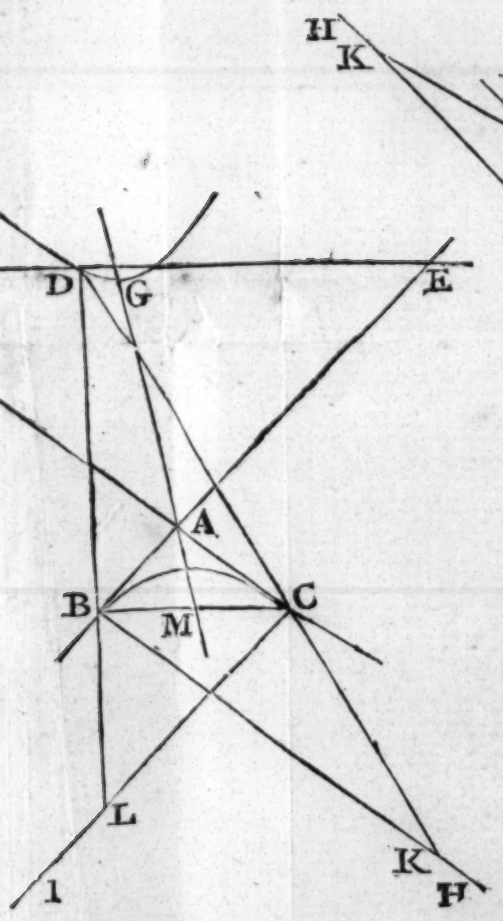
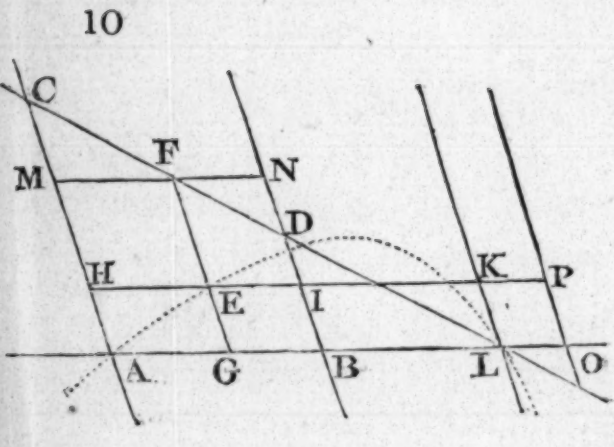
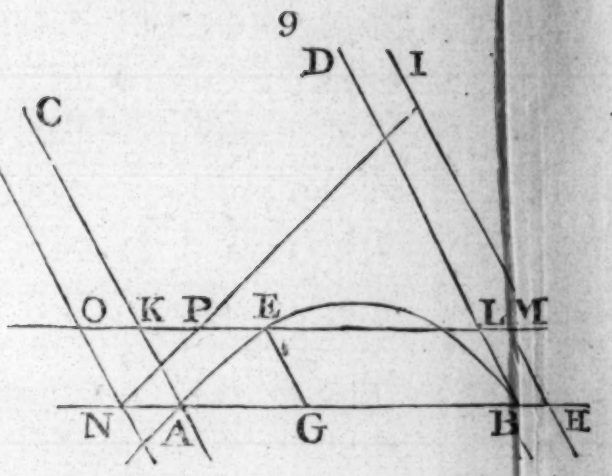
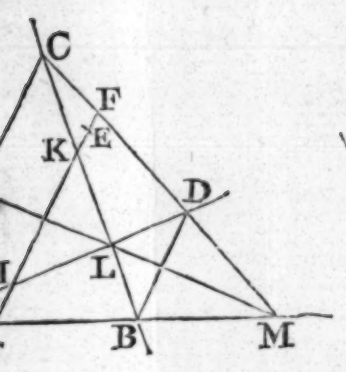
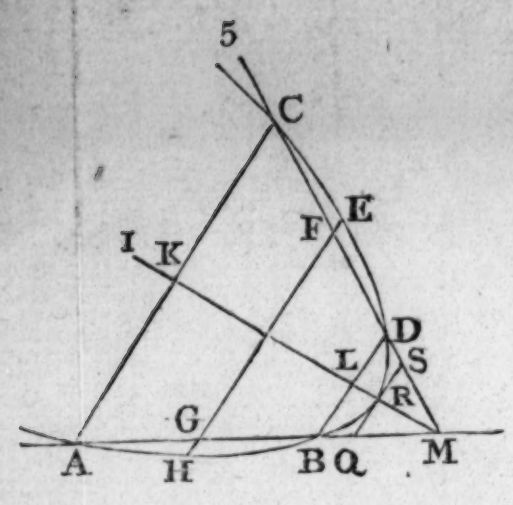
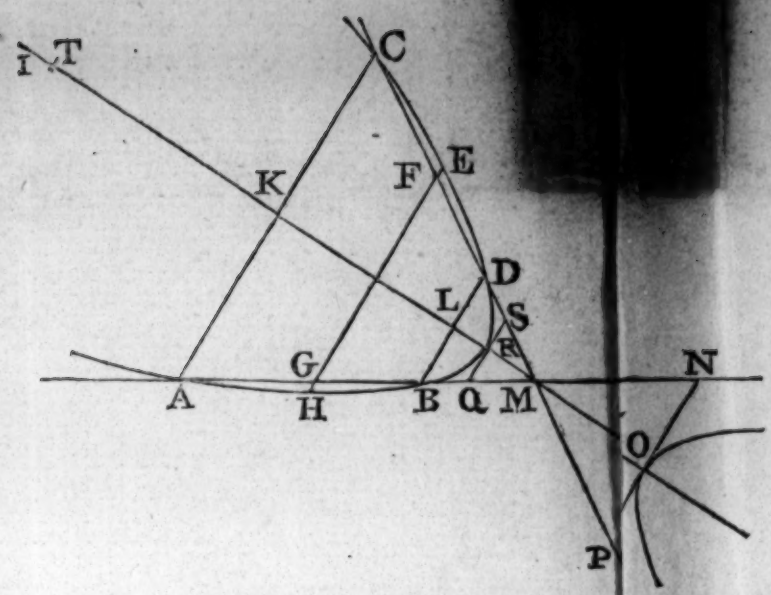
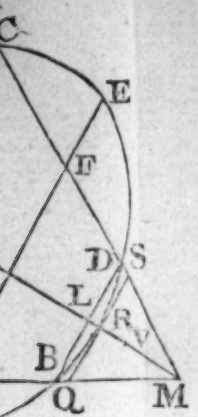
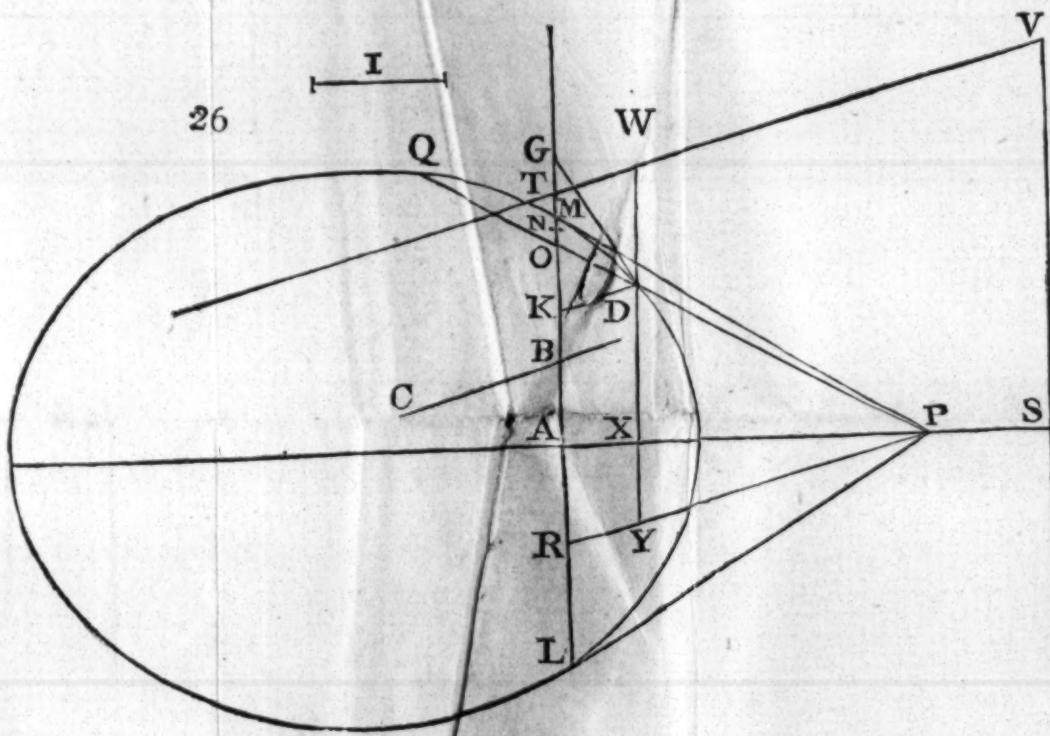
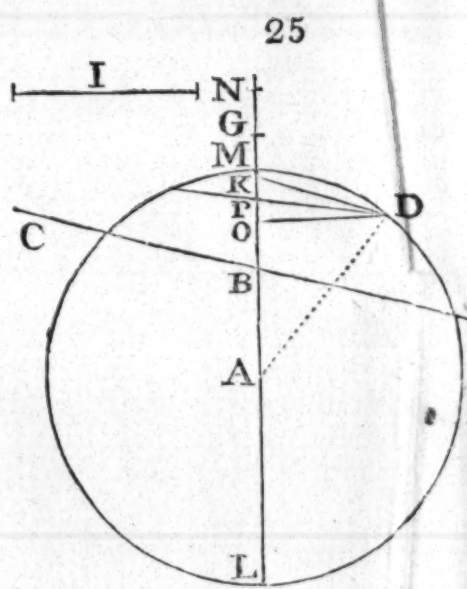
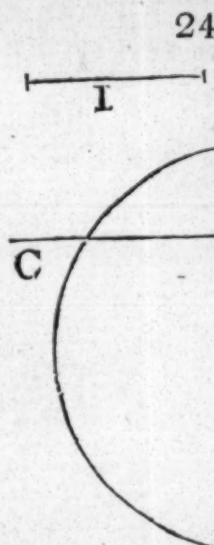
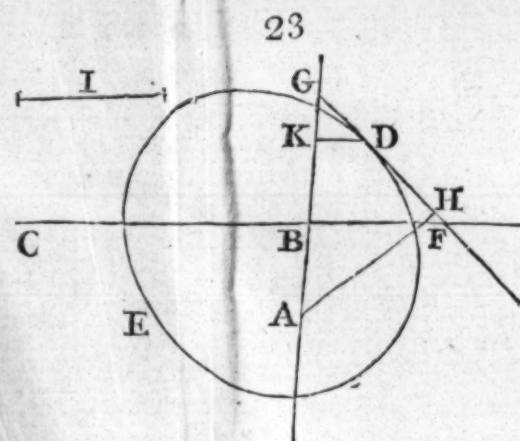
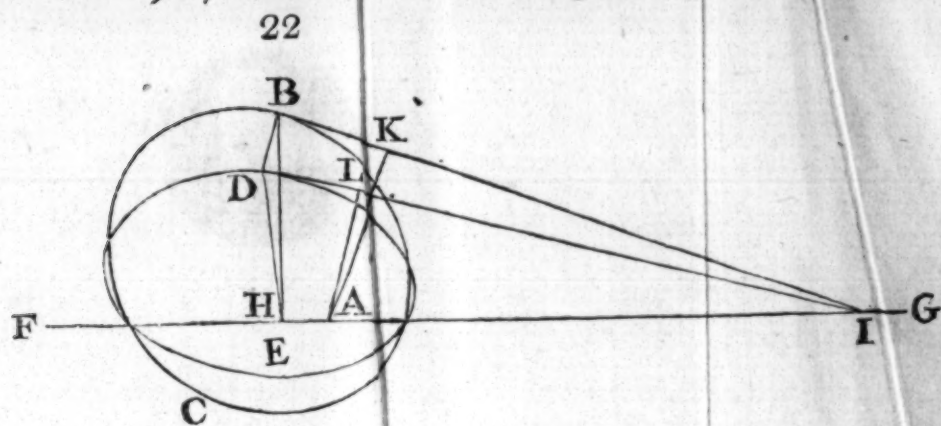
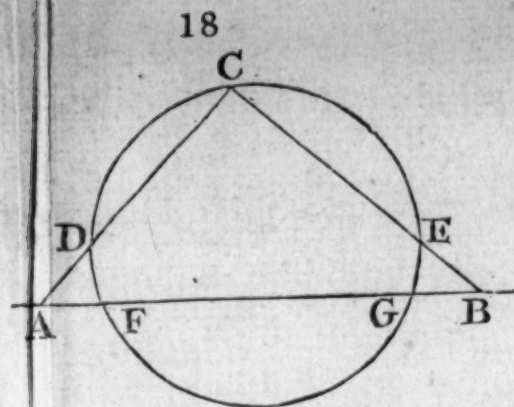
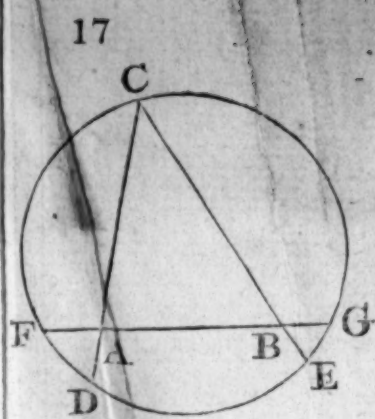
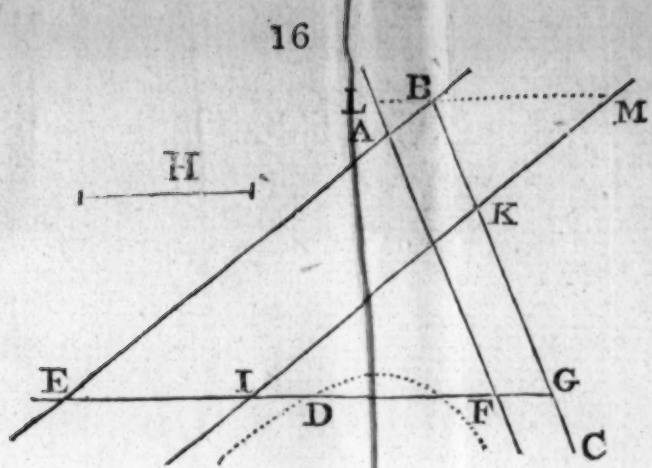
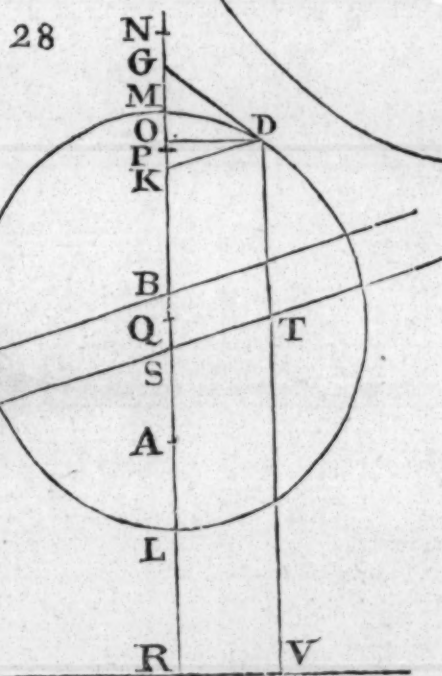
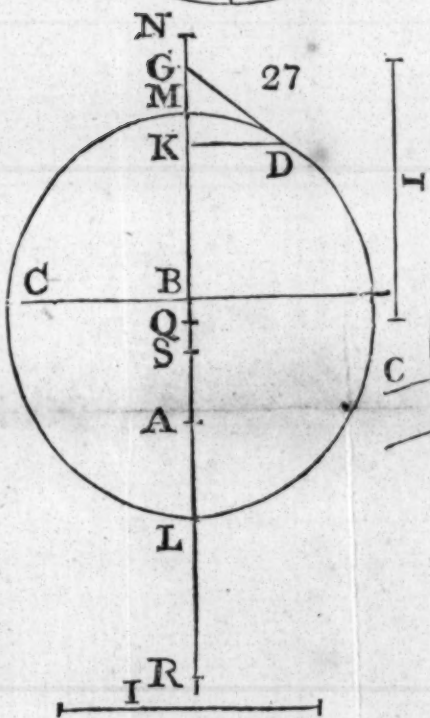
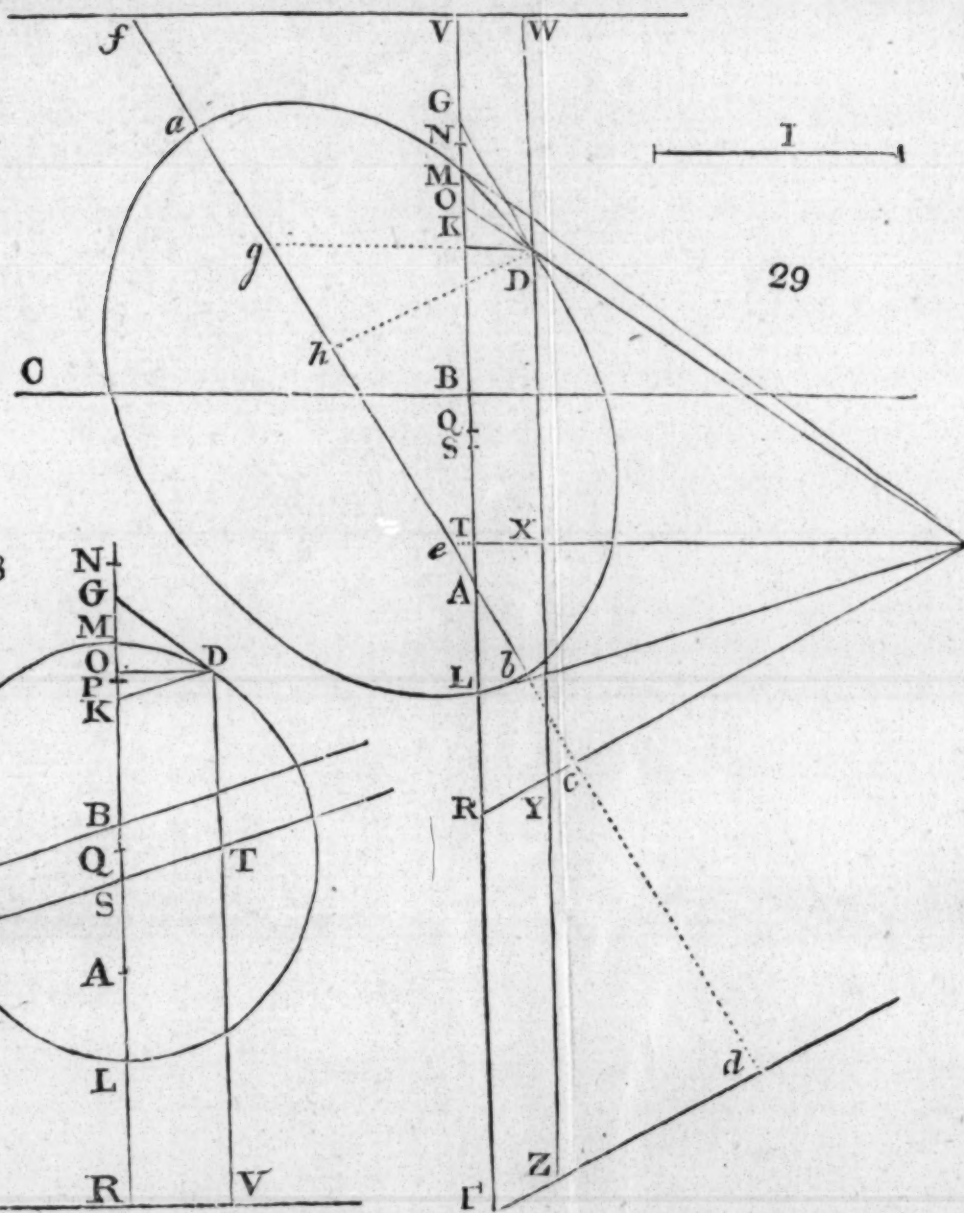
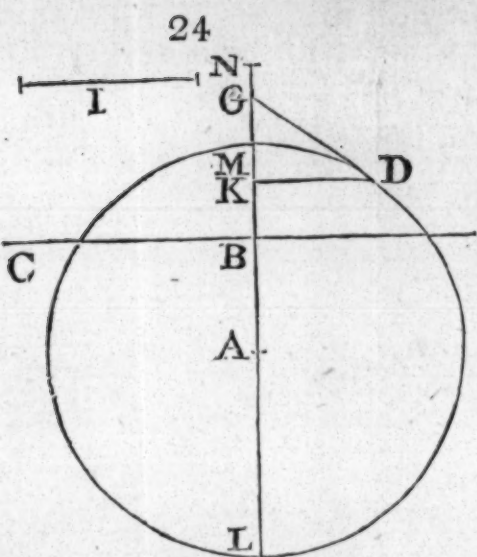
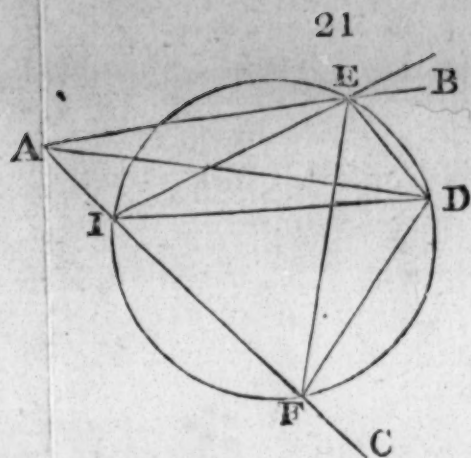
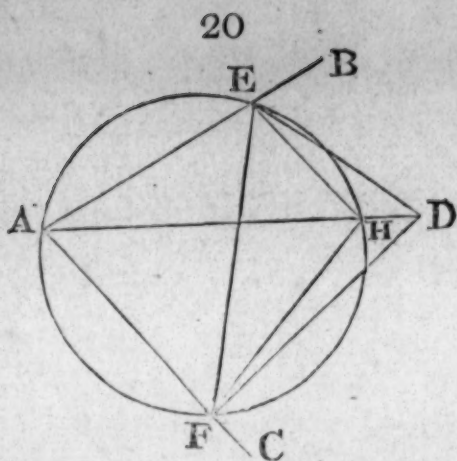
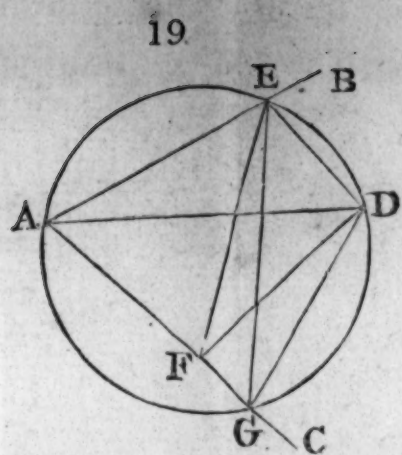
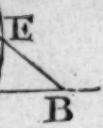


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the same purpose: the last rule, for computing the distance of the Moon from a star, though only an approximation, being so very exact, seems particularly adapted for the construction of a nautical Ephemeris, containing the distances of the Moon from the Sun and proper fixed stars ready calculated for the purpose of finding the longitude from observations of the Moon at sea; an assistance which, in an age abounding with so many able computers, mariners need not doubt they will be provided with, as soon as they manifest a proper disposition to make use of it.

A RULE

To compute the contraction of the apparent distance of any two heavenly bodies by refraction; the zenith distances of both, and their distance from each other being given nearly.

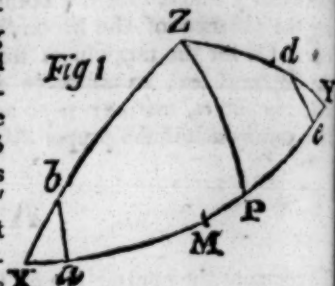
Add together the tangents of half the sum, and half the difference of the zenith distances; their sum, abating 10 from the index, is the tangent of arc the first. To the tangent of arc the first, just found, add the cotangent of half the distance of the stars; the sum, abating 10 from the index, is the tangent of arc the second. Then add together the tangent of double the first arc, the co-secant of double the second arch, and the constant logarithm of 114" or 2,0569: the sum abating 20 from the index, is the logarithm of the number of seconds required, by which the distance of the stars is contracted by refraction: which therefore added to the observed distance gives the true distance cleared from the effect of refraction.

Explication of the foundation of the preceding rule.

This rule is founded upon an hypothesis that the refraction in altitude is as the tangent of the zenith distance: and the refraction at the altitude of 45 degrees being 57", according to Dr. Bradley's observations, therefore the refraction at any altitude, calling the radius unity is $= 57'' \times \text{tangent of the zenith distance}$. This rule is exact enough for the purpose of the calculation of the longitude from observations of the distance of the Moon from stars at sea as low down as the altitude of 10° , for there the error is only 10" from the truth. But, if the altitude of the Moon or star be less than 10° , the rule may be still made to answer sufficiently, by only first correcting the observed zenith distances by subtracting from them three times the refraction corresponding to them taken out of any common table of refraction, and making the computation with the zenith distances thus corrected. This correction depends upon Dr. Bradley's rule for refraction, which he found to answer, in a manner exactly, from the zenith quite down to the horizon, namely that the refraction is $= 57'' \times \text{tangent of the apparent zenith distance lessened by three times the corresponding refraction taken out of any common table}$.

Demonstration of the preceding Rule.

Let ZXY represent a spherical triangle formed by great circles joining the zenith Z and the stars X and Y . Refraction acting in the vertical circles ZX and ZY will carry the star X nearer the zenith by a quantity $Xb = 57'' \times \text{tangent of } ZX$, and the stars Y towards Z by the quantity $Yd = 57'' \times \text{tangent of } ZY$; so that the apparent distance of the two stars will be bd instead of XY or less than XY , the true distance, by the Sum of the two little



spaces Xa , Yc , terminated by the perpendiculars ba and de . The little space $Xa = Xb \times \text{cofine of the angle } ZXY$ (calling radius unity) $= 57'' \times \text{tang. of } ZX \times \text{cofine of angle } ZXY$, or, by spherics, $= 57'' \times \text{tang. of } XP$. (ZP being an arch drawn from Z perpendicular to the arc XY). In like manner the little space $Yc = 57'' \times \text{tang. of } YP$; and therefore $Xa + Yc$ or the total effect of refraction $= 57'' \times \text{tang. } XP + \text{tang. } YP$. Let M be the middle of the arch XY , and put the tangent of XM or YM or $\frac{1}{2} XY = t$, and the tangent of MP or the distance of the perpendicular ZP from M the middle of the arch

$XY = n$. By trigonometry, $\text{tang. of } XP$ or $XM + MP = \frac{t+n}{1-n}$

and $\text{tang. of } YP$ or $YM - MP = \frac{t-n}{1+n}$ the sum of which,

$$\text{tang. } XP + \text{tang. } YP = \frac{t+n}{1-n} + \frac{t-n}{1+n} = \frac{2t+2t}{1-t^2n^2} = \frac{2tn}{1-t^2n^2} \times \frac{1+n^2}{2n} \times 2. \text{ Now } \frac{2tn}{1-t^2n^2} \text{ is the tangent of double}$$

the angle whose tangent is tn , and tn or the product of the tangents of XY and MP , by spherics, is equal to the product of the tangents of half the sum and half the difference of the zenith distances ZX and ZY .

whence $\frac{2tn}{1-t^2n^2}$ is equal to the tangent of double arch the first found by the rule. Also arch the second found by the rule being by spherics

MP , whose tangent is represented by n , and $\frac{1+n^2}{2n}$ being by trigonometry equal to the cosecant of double the arch whose tangent is n , therefore $\frac{1+n^2}{2n} = \text{cosecant of } 2MP$ or double arch the second. Whence

rule is manifest; namely, that $Xa + Ye$, the total effect of refraction in contracting the apparent distance of the two stars $= 57'' \times \tan. \angle P + \tan. YP = \text{double } 57'' \text{ or } 114'' \times \text{tang. of double the first arch} \times \text{cofecant of double the second arch. Q. E. D.}$

REMARK.

When the perpendicular arch ZP , let fall from the Zenith on the arch XY , falls without the triangle ZXY , the effect of refraction in diminishing the apparent distance of the stars X, Y is the difference of Xa and Ye : but the rule being general, gives always the sum or difference, whichever it be, which is a great advantage, and removes all grounds of ambiguity in the correction of refraction; as the total effect thereof always diminishes the distance of two stars from each other, however they are posited.

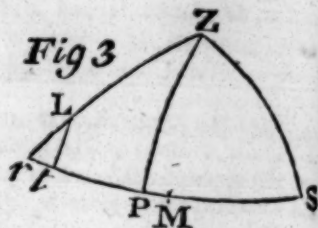
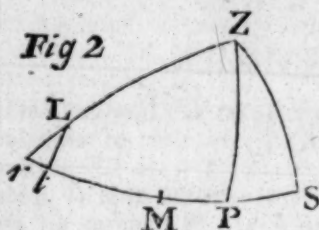
A RULE

To compute the contraction or augmentation of the apparent distance of the Moon from a star, on account of the Moon's parallax; the zenith distances of the Moon and star, and their distance from each other being given nearly.

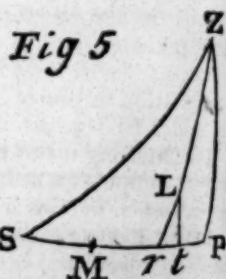
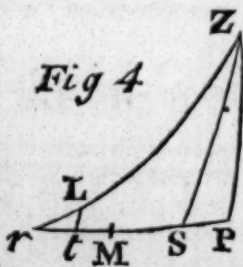
Add together the tangents of half the sum, and half the difference of the zenith distances of the Moon and star, and the cotangent of half the distance of the Moon from the star; the sum, abating 20 from the index, is the tangent of an arch, which call A . Then, if the zenith distance of the Moon is greater than that of the star, take the sum of the arch A , just found, and half the distance of the Moon from the star; but, if the zenith distance of the Moon be less than that of the star, take the difference of the said arch A and half the distance of the Moon from the star; and the sum, or difference call B . To the tangent of B , thus found, add the cosine of the Moon's zenith distance, and the logarithm of the Moon's horizontal parallax, expressed in minutes and decimals; the sum, abating 20 from the index, is the logarithm of the effect of parallax, tending always to augment the apparent distance of the Moon from the star; except the zenith distance of the Moon be less than that of the star, and, at the same time, the arch A be greater than half the distance of the Moon from the star, in which case the effect of parallax diminishes the apparent distance of the Moon from the star.

DEMONSTRATION.

In the spherical triangle ZLS, see Fig. 2, 3, 4, and 5th, Z represents the zenith, L the Moon, and S



the star; the effect of parallax depressing the Moon from L to r , r is the apparent place of the Moon, and rS the apparent distance of the Moon from the star; let fall the perpendicular Lt upon rS , produced if necessary, and rt will be the difference of LS and rS , or the effect of parallax.



Draw the arch ZP perpendicular to rS , and let M be the middle of rS . The Moon's parallax in altitude, being to her horizontal parallax, as the sine of her apparent zenith distance, to the radius, $Lr = \text{Moon's horizontal parallax} \times \sin$ of Zr ; and rt the effect of parallax upon the apparent distance of the Moon from the star will be $= Lr \times \cos$ of $ZrS = \text{horizontal parallax} \times \sin$ of $Zr \times \cos$ of ZrS (or, because $\tan. rP : \cos. ZrP :: \tan. Zr : \text{rad} :: \sin. Zr : \cos. Zr$; and therefore $\sin. Zr \times \cos. ZrS = \cos. Zr \times \tan. rP = \text{horizontal parallax} \times \cos. Zr \times \tan. rP$ agreeably to the rule. For it is evident by spherics that the arch A , found by the rule, is the same with MP the distance of the perpendicular from the middle of the arch rS : and it is evident, by the inspection of the figures, that the arch B or rP is equal to the sum of rM and MP , if the zenith distance of the Moon be greater than that of the star, as in Fig. 2d and 4th; but is the difference of rM and MP , if the zenith distance of the Moon be less than that of the star, as in Fig. 3d and 5th. Lastly, it may appear from the consideration of the figures, that, as the effect of parallax depresses the Moon directly towards the horizon, so it will always encrease her apparent distance from a star, except in the case represented by Fig. 5th; that is to say, unless the zenith distance of the Moon be less than that of the star, and, at the same time, the arch MP be greater than rM or half the distance of the Moon from the star. Q. E. D.

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Remarks on the use of the two foregoing rules.

It has been remarked, after the rule for refraction above, that if the altitudes of the Moon or star are under 10 degrees, the zenith distances must be first lessened by 3 times the refractions corresponding to their respective altitudes before the effect of refraction be computed.

But in order to compute the effect of parallax from the second rule, the observed distance of the Moon from the star must be first corrected by adding the effect of refraction to it found by rule the first; as must the observed altitudes of the Moon and star be also corrected by taking from them their respective refraction in altitude, and the corrected arches thus found must be made use of in computing the parallax. Only, if the altitude of the Moon and star are both 10 degrees or more, part of the calculation of rule the second may be saved, and arch the second, found by rule the first, taken for arch A in the second rule without any sensible error. In this case, it will be most convenient to observe the following order of computation instead of that before prescribed to be used when the altitudes are under 10 degrees.

1st. Making use of the apparent altitudes of the Moon and star uncorrected, compute arches the first and second by the directions contained in the rule of refraction.

2^{dly}. Taking arch the second for arch A in the rule of parallax, compute the effect of parallax according to rule the second.

3^{dly}. With arches the first and second compute the effect of refraction by rule the first.

4^{thly}, and lastly. Applying the two corrections of parallax and refraction duly, according to the rules, to the observed distance of the Moon from the star, you will have the true and correct distance of the Moon from the star, cleared both of refraction and parallax.

A RULE

For computing a second, but smaller correction than the first, necessary to be applied to the observations of the distance of the Moon from a star on account of parallax.

Call the principal effect of parallax, found by the preceding rule, the parallax in distance; and find the parallax answering to the Moon's altitude. Then to the constant logarithm 0.941 add the logarithm of the sum of the parallax in altitude and the parallax in distance, the logarithm of the difference of the same parallaxes, and the cotangent of the observed distance of the Moon from the star (corrected for refraction, and the principal effect of parallax), the sum, abating 13 from the index, is the logarithm of the number of seconds required, being the second correction of parallax; and is always to be added to the distance of the Moon from the star, first corrected for refraction, and the principal effect of parallax found above, in order to obtain the true distance; unless the distance exceeds 90 degrees, in which case it is to be subtracted.

DEMONSTRATION.

Let *L* Fig. 6. represent the Moon's true place in the sphere, and *r* her apparent place as depressed by parallax, *S* the place of the star, and *Lt* a perpendicular let fall from the true place of the Moon *L* upon the great circle *rS* joining the star *S* and the apparent place of the Moon *r*; (all as in the four figures belonging to the preceding rule). Let *L a* be the arch of a parallel circle described from the star *S* as a pole through the true place of the Moon *L*. *S a* terminated by the parallel circle *L a*, and not *St* terminated by the perpendicular *Lt*, as was supposed in the former demonstration, is equal to *SL* or the true distance of the Moon from the star, which was therefore computed too small from the former rule by the little space *at*. Let *LT* and *aT* be the equal tangents of the equal arches *LS* and *aS* in *L* and *a*, meeting in the radius *CS*, drawn from the center of the sphere *C* and produced, in *T*. The space *L a t*, on account of its smallness, may be looked upon as lying all in one plane, namely *L a T*, and *L a* as the small arch of a circle described from the point *T* as a centre with the line *LT* as a radius, through *L* and *a*, *L a* as the sine, and *at* as the versine of the arch *L a*; and consequently *at* equal to the square of *Lt* divided by *2 LT*. But the triangle *Lrt* being right-angled in *t*, the square of *Lt* is equal to the difference of the squares of *Lr* and *rt*, and consequently to the product of their sum and difference; that is to say,

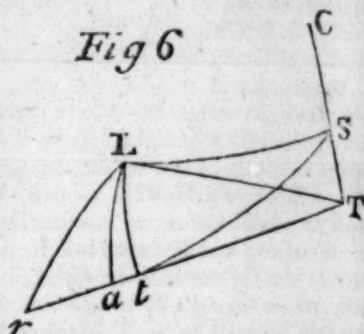
$$at = \frac{Lr + rt \times Lr - rt}{2 LT} \text{ or (because the tangent } TL \text{ is equal to}$$

$$\text{the square of the radius } CS \text{ divided by the cotangent of } LS) \\ = \frac{Lr + rt \times Lr - rt}{2 \times \text{square of } CS} \times \text{cotangent of } LS.$$

Now suppose the spaces *Lr*, *rt* to be expressed in minutes, which will be most convenient in practice, then the radius of the sphere *CS* must be taken equal to $3437\frac{3}{4}$, for so many minutes are contained in an arch of a circle equal to its radius:

$$\text{and } at \text{ will be } = \frac{Lr + rt + Lr - rt \times \text{cotan. of } LS}{2 \times 3437\frac{3}{4} \times 3437\frac{3}{4}}. \text{ But,}$$

the cotangents of similar arches of circles of different radii being directly as the radii, therefore the cotangent of *LS* to the radius *CS* or $3437\frac{3}{4}$, is to the cotangent of the same arch to 10000000000, which is the radius to which the logarithmic tables are adapted, its logarithm being 10; as $3437\frac{3}{4}$ to 10000000000. Therefore the cotangent of *LS* = tabular cotangent of *LS* $\times 3437\frac{3}{4}$, which, being substituted in the value of *a*



above, gives at , expressed in minutes $= \frac{Lr+rt + Lr-rt \times \text{tabu-}}{68755000000000}$

lar cotangent LS or, multiplying by 60, the value of at will come

out in seconds $= \frac{Lr+rt \times Lr-rt \times \text{tabular cotangent of } LS}{1146000000000}$

The logarithm of the denominator 1146000000000 is 12,059, instead of subtracting which, when the operation is performed by logarithms, add 0,941 (its compliment to 13) to the value of the numerator found in logarithms, and subtract 13 from the index: the remainder will be the value of at in seconds. Q. E. D.

A concise rule to find the distance of the Moon from a zodiacal star, very nearly; the difference of the longitudes of the Moon and star, and the latitudes of both being given.

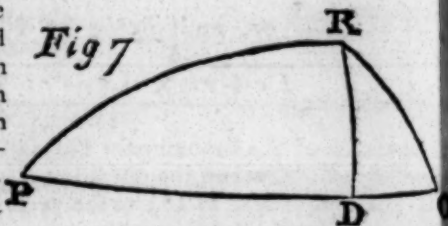
To the cosine of the difference of the longitudes add the cosine of the difference of the latitudes, if both of the same denomination, or sum; if of contrary denominations, the sum of the two logarithms, abating 10 from the index, is the cosine of the approximate distance. This gives the true distance of the Moon from the Sun, being then nothing more than the common rule for finding the hypotenuse of a right-angled spherical triangle from the two sides given. But in the case of a zodiacal star apply the following correction to the approximate distance thus found.

To the constant logarithm 5.3144 add the sine of the Moon's altitude, the sine of the star's latitude, the verse-sine of the difference of longitude, and the cosecant of the approximate distance; the sum of these 5 logarithms, abating 40 from the index, is the logarithm of a number of seconds, which subtracted from the approximate distance, found before, if the latitude of the Moon and star are of the same denomination, or added thereto, if they are of different denominations, gives the true distance of the Moon from the star.

N. B. This rule, though only an approximation, is so very exact, that even, if the latitude of the Moon was 5° , and that of the star 15° , the error would be only $10''$; and if the latitude of the Moon be 5° , and that of the star 10° , the error is only $4'' \frac{1}{2}$; and if the latitudes be less, will be less in proportion as the squares of the sines of the latitudes decrease.

DEMONSTRATION.

Let P, Fig. 7. represent one of the poles of the ecliptic, and Q, R the places of the Moon and star. From R let the arch of a great circle RD be drawn perpendicular to PQ. By spherics, the tangent of PD = tangent of PR $\times$ cosine of the angle RPD. And, by trigonometry, cosine of QD or (QP - PD) = cos. QP $\times$ cos. PD + fin. QP $\times$ fin. PD = cos. QP $\times$ cos. PD + fin. QP $\times$ cos. PD $\times$ tan. PD =



cos. PD $\times$ cos. QP + fin. QP $\times$ tan. PD = cos. PD $\times$ cos. QP + fin. QP $\times$ tan. PR $\times$ cos. P, cos. QD : cos. PD :: cos. QP + fin. QP $\times$ tan. PR $\times$ cos. P : 1. But, by spherics, cos. QD : cos. PD :: RQ : cos. PR :: cos. RQ : cos. PR :: cos. QP + fin. QP $\times$ tan. PR $\times$ cos. P : 1. Whence cos. RQ = cos. PR $\times$ cos. QP + fin. QP $\times$ tan. PR $\times$ cos. P : Now, by trigonometry, cos. (QP - PR) = cos. QP $\times$ cos. PR + fin. QP $\times$ fin. PR ; whence fin. QP $\times$ fin. PR = cos. (QP - PR) - cos. QP $\times$ cos. PR ; which being substituted above, gives cos. RQ = cos. (QP - PR) $\times$ cos. P - cos. PR $\times$ cos. QP $\times$ cos. P + cos. PR $\times$ cos. QP = cos. (QP - PR) $\times$ cos. P + verse-fine P $\times$ cos. PR $\times$ cos. QP. Now put cos. (QP - PR) $\times$ cos. P = cos. G, or the approximate distance, then cos. RQ = cos. G, or (because the difference of RQ and G is but small) $G - RQ \times \text{fin.} \left(\frac{G + RQ}{2} \right) = \text{verse-fine P} \times \text{cos. PR} \times \text{cos. QP}$ nearly. Whence $RQ = G - \frac{\text{verse-fine P} \times \text{cos. PR} \times \text{cos. QP}}{\text{fin.} \frac{G + RQ}{2}}$ nearly = $G - \frac{\text{v. f. P} \times \text{cos. PR} \times \text{cos. QP}}{\text{fin. G}}$ nearly. Q. E. D

Note, the error of this formula arises from taking $G = \frac{G + RQ}{2}$ which means it will always give RQ too great, nearly by the following quantity $\frac{1}{2} Sg \text{ cos. PR} \times Sg \text{ cos. QP} \times \cot. G \times Sg \text{ tan.} \frac{1}{2} G$. This comes to a maximum when G is 60° , and is then $= \frac{1}{6} \sqrt{\frac{1}{3}} \times Sg \text{ cos. PR} \times Sg \text{ cos. PQ}$. If the latitudes of the Moon and star are both 5° it is $= 1''$. If the Moon's latitude be 5° , and that of the star 10° it is $= 4''$ and if the latitude of the star be 15° it is $= 10''$.

FINIS.

II

Q

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PD=
QP +
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PD
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(R)
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PR ×
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= 4